Job Matching and Affirmative Action: The Impact of Transfer Policies*

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Abstract

Subsidies and taxes are widely used in labor markets to shape employment outcomes. This paper studies how such transfers affect the welfare of minority workers in stable allocations within a Kelso–Crawford framework. Although affirmative action policies are often designed to improve labor market opportunities, we show that they may unintentionally reduce minority welfare. Our first main result establishes that only uniform transfers—those providing the same subsidy or tax to all members of a group across firms—guarantee that minority welfare is never reduced. Beyond this, we derive conditions under which transfers can strictly increase the utility of targeted workers, reduce unemployment, or improve the representation of minority workers across firms, without harming others. These results clarify the limits of targeted interventions and offer implications for the design of affirmative action policies in assignment markets.

Keywords: Job matching; Salary; Transfer policy; Market equilibrium; Subsidy; Taxation; Affirmative action
JEL Classification: C78; D47; D50; J20; J30

1 Introduction

Subsidies and taxes are widely used policy instruments in labor markets to promote employment and reduce inequalities. In the United States, firms receive tax abatements for hiring at least a minimum number of minority workers (Byrnes et al. 1999; Slattery and Zidar 2020). In many OECD countries, employment subsidies cover part of the salaries of disadvantaged workers (Snower 1994). However, such policies can induce complex responses in firms' hiring and wage-setting behavior and, in some cases, may unintentionally reduce the welfare of minority workers in stable allocations.[1] By increasing a worker's attractiveness, subsidies intensify competition among workers,

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[1] Kojima (2012) shows that affirmative action policies harm minority students in a model without transferable utility, for any stable mechanism.

thereby affecting both assignments and salaries. While both are key determinants of worker welfare, most of the existing literature focuses on assignment, leaving salaries underexplored. This paper addresses this gap by studying how transfer policies affect stable allocations and worker welfare.

We develop a matching model with transferable utility and study the effects of affirmative action policies on stable allocations. Workers' *utilities* depend on both their *assignments* and the *salaries* they receive. Firms are endowed with *revenue functions*, which specify the revenue generated by any set of workers. Their objective is to maximize *profits*, defined as the sum of the firm's revenue from its workers plus the total transfers—subsidies or taxes—received for each worker-firm pair, minus the salaries paid to workers. A firm's *demand* consists of the sets of workers that maximize profits given prevailing salaries and transfers. Within this framework, the *gross substitutes condition* ensures the existence of stable allocations, and these allocations coincide with the set of competitive equilibria (Shapley and Shubik 1971; Kelso and Crawford 1982; Gul and Stacchetti 1999; Kojima et al. 2024).[2] Moreover, this condition ensures the existence of two stable allocations that maximize the objectives of each side of the market: the *firm-optimal stable allocation*, in which firms maximize profits, and the *worker-optimal stable allocation*, in which workers maximize utilities. For each market, we consider a *mechanism* leading to each of these allocations.

To analyze the impact of affirmative action policies on minority workers' welfare and market assignments, we consider a partition of the workforce into a *minority* group and a *majority* group.[3] A natural requirement is that, when a policy is introduced, no minority worker is worse off. If this condition holds for the allocations induced by a mechanism, we say that a transfer *does not reduce the welfare of minority workers* under that mechanism; otherwise, the transfer *reduces the welfare of minority workers*. Our first main result establishes that *uniform transfers*—those providing the same subsidy or tax to every member of a group across all firms—are necessary and sufficient to guarantee that minority welfare is never reduced under both the firm-optimal and worker-optimal stable mechanisms. Moreover, this conclusion extends to the aggregate welfare of minority workers, for which uniform transfers are again necessary and sufficient to prevent any reduction in total utility. Considering the firm-optimal stable mechanism, we further show that any tax on majority workers never reduces the welfare of minority workers. By contrast, only uniform transfers guarantee that minority welfare is never reduced under the worker-optimal stable mechanism.

In practice, many policies are targeted toward subsets of minority workers, for instance by providing subsidies to the unemployed or to workers in specific sectors. Our first result indicates that such differentiated transfers can reduce minority welfare. Workers receiving smaller or no subsidies lose bargaining power, potentially leading to lower salaries or even job loss. Firms may prefer subsidized

[2]The gross substitutes condition requires that a set of demanded workers remains demanded following an increase in other workers' salaries. This assumption is widely adopted for both technical and practical reasons. Cramton et al. (2006), Bichler and Goeree (2017), and Milgrom (2017) discuss how market design incorporating the gross substitutes condition is effectively addressed, whereas its absence leads to unavoidable trade-offs, such as between efficiency, incentives, and complexity properties. Moreover, this condition is particularly relevant for specialized markets, such as the labor market for gastroenterologists (Niederle and Roth 2003), in situations where resources are relatively homogeneous (e.g., unskilled labor), when resources are prepackaged to reduce complementarity (Loertscher et al. 2015), and when complementary resources are fixed in the short term (e.g., rental markets for agricultural land that require specific machinery investments).

[3]The terms minority and majority workers refer to socioeconomic characteristics and do not impose restrictions on group sizes or on the distribution of revenue generated within firms.

minority workers over others who are already employed but receive no subsidy. While uniform transfers prevent these adverse effects, they limit policymakers' ability to target specific groups. In particular, uniform transfers do not guarantee an improvement in minority welfare, which is often the primary objective of these policies.

Our second main result establishes a sufficient condition under which a subsidy strictly increases the utility of targeted workers at the worker-optimal stable allocation. This condition requires that the (weakly) highest subsidy be directed to the firm currently employing the worker. In this case, the additional surplus generated by the subsidy is fully captured by the worker, without altering the assignment or reducing the welfare of others. Notably, this policy can also increase the salaries of non-targeted workers, including majority workers, by alleviating competition among workers. Implementing such a transfer requires knowledge of the current assignment and of the fact that the market is at the worker-optimal stable allocation, since applying it at the firm-optimal stable allocation could reduce minority welfare.

Minority groups can be defined along multiple dimensions, including gender, location, or ethnicity. When policies target only a subset of these groups, some minority workers may benefit while others do not. Our analysis shows that this situation is equivalent to implementing a non-uniform transfer, which can reduce the welfare of certain minority workers even when a uniform transfer is introduced. We identify two solutions to mitigate these negative effects. The first is to treat all minority workers identically through a uniform transfer. The second is to impose a uniform tax on majority workers, which ensures that the welfare of the rest of the population is not reduced. However, the effects of such taxes cannot be targeted to specific groups and may benefit workers who are not members of a minority group. These findings highlight both the importance and the challenges of designing transfers in labor markets with multiple minority groups.

Affirmative action policies are also used to modify the representation of minority workers in the market. We study two objectives: *increasing the representation of minority workers* and *promoting the employment of unemployed minority workers*. To analyze these interventions, we focus on a subclass of markets with transferable utility in which firms' revenue functions are *additively separable* and subject to *capacity constraints*. In this subclass, each worker is treated individually, so that the revenue from any set of workers that satisfies the capacity constraint equals the sum of the revenues they generate individually. We identify transfers that achieve representation goals without reducing minority welfare. To limit the cost of these policies, we further analyze the substitutability between subsidies and taxes, showing that a uniform subsidy to minority workers yields the same market allocation as an equivalent tax on majority workers. These findings have important implications for market design, which we discuss in Section 6.

Related Literature

A large literature in market design examines the existence of stable assignments, or competitive equilibria (Kelso and Crawford 1982), under affirmative action policies. In non-transferable utility (NTU) markets, these contributions identify the conditions under which stable assignments exist despite representation constraints such as minimum guarantees (or lower quotas), upper quotas,

and regional quotas (Abdulkadiroğlu and Sönmez 2003; Kamada and Kojima 2012, 2015, 2017, 2024, Ehlers et al. 2014; Kominers and Sönmez 2016; Sönmez et al. 2019, among others). In transferable utility (TU) markets, Gul et al. (2019), Kojima et al. (2020), 2024, and Echenique et al. (2021) identify conditions under which a competitive equilibrium exists when such constraints are imposed. While these studies ensure minority representation in firms, they often overlook how such policies affect the welfare of minority workers. In contrast, our paper investigates the impact of affirmative action policies on minority workers' welfare. In addition, we show that, without imposing quotas, it is possible to incentivize firms to achieve targeted minority representation through the use of taxes and subsidies. Thus, our paper provides insights into the subsidies required to induce firms to achieve the desired minority representation in stable allocations.

Despite the similarities between TU and NTU markets, important distinctions exist. While Ehlers et al. (2014) show that imposing floor constraints can lead to an empty set of stable matchings in NTU markets, Kojima et al. (2020) demonstrate that, under *generalized interval constraints*, the substitutes condition is preserved in TU markets, ensuring the existence of a stable allocation. Similarly, Kojima (2012) establishes that it is impossible in NTU markets to design affirmative action policies that guarantee no harm to minority agents under any stable mechanism. By contrast, we show that in TU markets, uniform transfers within groups never reduce the welfare of minority workers. This distinction is discussed in Appendix A.1. The key intuition is that the continuity of transfers in TU markets allows changes in agent attractiveness to affect both assignments and salaries. A tax or subsidy can alter relative desirability and thereby influence salaries, even if the assignment itself remains unchanged. In NTU markets, welfare depends solely on the assignment: modifying the desirability of some agents has no effect as long as ordinal rankings representing preferences remain unchanged. Transfers in TU markets capture the competitive interactions present in the market by accounting for alternative opportunities available to agents—something not reflected in simple ordinal rankings. To the best of our knowledge, this is the first paper to show that affirmative action policies can reduce minority welfare in TU markets, and to identify the precise conditions under which such adverse effects are avoided.

In recent years, affirmative action policies have increasingly aimed not only to reduce unemployment among minority workers but also to raise their salaries and improve their representation across firms and sectors (Yang and Murali 2024; Chen et al. 2025). These objectives, however, have received limited attention in the market design literature. Our theoretical analysis contributes to filling this gap and complements a growing empirical literature on the effects of affirmative action on both representation and welfare. For instance, Bleemer (2022) documents that affirmative action in education may reduce labor market earnings for minorities, while studies such as Leonard (1990) and Coate and Loury (1993) highlight the limited compliance of firms with employment regulations, leading to persistent disadvantages for minority workers. In this context, our framework identifies transfers—implemented through subsidies and taxes—that align firm incentives with affirmative action goals. Specifically, we design transfers that achieve desired minority representation across firms, reduce unemployment, and raise minority salaries, while ensuring that the welfare of other minority workers is not reduced.

This paper proceeds as follows. Section 2 introduces the model. Section 3 investigates transfers that

never reduce the welfare of minority workers under optimal stable mechanisms. Section 4 examines the effects of transfers in markets with multiple minority groups. Section 5 explores common affirmative action policies, their associated costs, and how surplus is distributed. Section 6 discusses policy implications and highlights potential sources of welfare loss. Appendix A provides additional insights. Appendix B details the algorithms underlying the mechanisms discussed. Auxiliary results are presented in Appendix C, while all proofs are provided in Appendix D.

2 The Model

There are finite sets W and F of *workers* and *firms*. Agents on both sides of the market derive value from being matched to agents on the other side. Let $\sigma_{w,f} \in \mathbb{R}$ denote the *value* that worker w derives from being employed by firm f. Let $\boldsymbol{\sigma} \equiv (\sigma_{w,f})_{w \in W,\, f \in F}$ denote the *profile of workers' values*. Each worker can be employed by at most one firm. For each firm $f \in F$, we define a *revenue function* $R_f : 2^W \to \mathbb{R} \cup \{-\infty\}$, which maps a subset of workers to the revenue of firm f if it hires them.[4] Let $R \equiv (R_f)_{f \in F}$ denotes the *profile of firms' revenue functions*. We normalize the value that unemployed workers derive to zero, $\sigma_{w,\varnothing} = 0$, and for any subset $W' \subseteq W$, $R_\varnothing(W') = 0$, where $\varnothing$ denotes the *null firm* representing unemployment for workers. All workers have the option of remaining unemployed.

In markets, we allow for utility transfers between firms and workers, which we refer to as *salaries*. Let $s_{w,f} \in \mathbb{R}$ denote the salary received by worker $w \in W$ when working for firm $f \in F \cup \{\varnothing\}$. Since each worker can be employed by at most one firm, we can represent salaries as a *vector of salaries* $\mathbf{s}$. We assume that unemployed workers receive a salary of zero, $s_{w,\varnothing} = 0$, and that workers are indifferent to which other workers firms hire.

Each worker's *utility function* is strictly increasing and continuous in the salaries. Let $u_w(f, s_{w,f}) = \sigma_{w,f} + s_{w,f}$ be the utility function of worker w when working for firm f at salary $s_{w,f}$. Consequently, $u_w(f, -\sigma_{w,f}) = 0$ and $-\sigma_{w,f}$ is the lowest salary at which worker w would consider working for firm f.[5]

A policymaker regulates the market using *transfers*, which represent *subsidies* or *taxes*. A transfer is a matrix $\mathbf{T} \equiv (t_{w,f})_{w \in W, f \in F}$, where each element $t_{w,f} \in \mathbb{R}$ represents the transfer for worker w when employed by firm f. If $t_{w,f} < 0$, we say that $\mathbf{T}$ *taxes* worker w at f by $t_{w,f}$, and if $t_{w,f} > 0$ we say that $\mathbf{T}$ *subsidizes* worker w at f by $t_{w,f}$. A *null transfer* is denoted by $\mathbf{T}_0$ such that for each worker w and each firm f, $t_{w,f} = 0$. Let $\mathbb{T}$ be the set of all possible transfers.

If a firm f hires a subset of workers $W' \subset W$ while facing a vector of salaries $\mathbf{s}$, a transfer $\mathbf{T}$, and a revenue function R_f, its *profit* is given by

$$V_f(W'; \mathbf{s}, R_f, \mathbf{T}) = R_f(W') - \sum_{w \in W'} s_{w,f} + \sum_{w \in W'} t_{w,f},$$

[4] We allow revenue functions to take the value $-\infty$ to capture infeasibilities. This approach is in line with recent contributions in trading networks (e.g., Hatfield et al. 2013; Fleiner et al. 2019; Schlegel 2022), where such formulations are used to model various restrictions. We adopt this construction to represent firm capacity in Section 5.

[5] In our model, salaries can be negative. This occurs when $\sigma_{w,f}$ is positive, meaning that worker w derives positive utility from working for firm f. In practice, this situation arises in the context of on-the-job training, where workers pay (or accept negative salaries) in exchange for acquiring skills that will increase their future employability.

that is, the firm's revenue minus salaries of workers it hires and the transfers received by the firm.[6] We define the *maximal profit function* as $\Pi_f(\cdot\,; R_f, \mathbf{T})$ and the *demand* by $D_f(\cdot\,; R_f, \mathbf{T})$ such that for each vector of salaries $\mathbf{s}$,

$$\Pi_f(\mathbf{s}; R_f, \mathbf{T}) = \max\{V_f(W'; \mathbf{s}, R_f, \mathbf{T}) : W' \subset W\};$$
$$D_f(\mathbf{s}; R_f, \mathbf{T}) = \{W' \subset W : V_f(W'; \mathbf{s}, R_f, \mathbf{T}) = \Pi_f(\mathbf{s}; R_f, \mathbf{T})\}.$$

Each element of $D_f(\mathbf{s}; R_f, \mathbf{T})$ is referred to as a *demand set*.
The *gross substitutes condition*, introduced by Kelso and Crawford (1982), ensures that if a worker's salary increases, firms retain their demand for workers whose salaries remain unchanged.

Definition 1 (Gross Substitutes)**.** A demand $D_f(\cdot\,; R_f, \mathbf{T})$ satisfies the gross substitutes condition if, for any two vectors of salaries $\mathbf{s}$ and $\mathbf{s}'$, with $\mathbf{s}' \geq \mathbf{s}$, and any $W' \in D_f(\mathbf{s}; R_f, \mathbf{T})$, there exists $W'' \in D_f(\mathbf{s}'; R_f, \mathbf{T})$ such that $\{w \in W' : s_w = s'_w\} \subseteq W''$.

A market is denoted by $G = (W, F, \boldsymbol{\sigma}, R, \mathbf{T})$. Since W, F, and $\boldsymbol{\sigma}$ are fixed throughout, and different markets correspond to variations in firms' revenue profiles, we write $G = (\mathbf{T})$ for simplicity. Let $\mathbb{G}$ be the set of all possible markets. We assume that in all markets, firms' demands satisfy the gross substitutes condition.
An *assignment* is a mapping (or correspondence) $\mu : W \cup F \rightrightarrows W \cup F \cup \{\varnothing\}$ satisfying:

- $\mu(w) \in F \cup \{\varnothing\}$,
- For any $w \in W$ and $f \in F$, $\mu(w) = f$ if and only if $w \in \mu(f)$, and
- $\mu(f) \subseteq W$.

We use the notation $\mu(w) = \varnothing$ to indicate that worker w is unemployed at μ. Given an assignment μ and a vector of salaries $\mathbf{s}$ we define an *allocation* as a pair $(\mu, \mathbf{s})$. With a slight abuse of notation, we write $u_w(\mu, \mathbf{s})$ for $u_w(\mu(w), s_{w,\mu(w)})$, and $V_f(\mu; \mathbf{s}, R_f, \mathbf{T})$ for $V_f(\mu(f); \mathbf{s}, R_f, \mathbf{T})$.
An allocation $(\mu, \mathbf{s})$ is *individually rational* if for each firm $f \in F$, $V_f(\mu; \mathbf{s}, R_f, \mathbf{T}) \geq 0$ and for each worker $w \in W, u_w(\mu, \mathbf{s}) \geq 0$. We now introduce the definition of *stability*.

Definition 2 (Stable Allocation)**.** An allocation $(\mu, \mathbf{s})$ is stable if it is individually rational, and there is no firm-worker set $\{f\} \cup W'$, with $W' \subseteq W$, and vector of salaries $\mathbf{s}'$, that *block* $(\mu, \mathbf{s})$ such that:

- $u_w(f, s'_{w,f}) \geq u_w(\mu, \mathbf{s})$, for each $w \in W'$, and
- $V_f(W'; \mathbf{s}', R_f, \mathbf{T}) \geq V_f(\mu; \mathbf{s}, R_f, \mathbf{T})$

with strict inequality holding for at least one member of $\{f\} \cup W'$.

We define *stable allocation optimality* in markets where firms' demands satisfy the gross substitutes condition. In such markets, there exists a stable allocation unanimously preferred by all workers, and another unanimously preferred by all firms. Without transfers, we denote the *worker-optimal*

[6] Subsidies and taxes are typically determined based on the firm's workforce. By adding the transfer to the profit, we assume the transfer is an *additively separable* function. Kojima et al. (2024) provides further discussion on transfers that preserve the substitutability condition.

stable allocation as $(\mu_W, \mathbf{s}_W)$ and the *firm-optimal stable allocation* as $(\mu_F, \mathbf{s}_F)$.[7] With a non-null transfer $\mathbf{T}$, we denote the corresponding worker-optimal stable allocation by $(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$ and the firm-optimal stable allocation by $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$.

A *mechanism* φ maps each market $G = (\mathbf{T}) \in \mathbb{G}$ to an allocation. The *worker-optimal stable mechanism* φ_W assigns to each market its worker-optimal stable allocation, $\varphi_W(\mathbf{T}) = (\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$, while the *firm-optimal stable mechanism* φ_F assigns the firm-optimal stable allocation, $\varphi_F(\mathbf{T}) = (\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$. A mechanism φ is stable if, for each market $G = (\mathbf{T}) \in \mathbb{G}$, the allocation $\varphi(\mathbf{T})$ is stable.

In this paper, we study the impact of affirmative action on groups of the population.[8] Mathematically, we fix a *partition* of W, denoted by $\mathcal{P} \subset 2^W \setminus \emptyset$, where $\bigcup_{P \in \mathcal{P}} P = W$, and $P \cap P' = \emptyset$ for $P \neq P'$. Each element of $\mathcal{P}$ is a *group*. Unless otherwise stated, we restrict the partition $\mathcal{P}$ to two groups: m, representing *minority workers*, and M, representing *majority workers*. In Section 4, we extend the analysis to partitions with more than two groups. A transfer $\mathbf{T}$ *taxes majority workers* if $t_{w,f} \leq 0$ for all $w \in W$ and $f \in F$, and $t_{w,f} < 0$ only if $w \in M$. Similarly, a transfer $\mathbf{T}$ *subsidizes minority workers* if $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$, and $t_{w,f} > 0$ only if $w \in m$.[9] A transfer *taxes majority and subsidizes minority workers* if $t_{w,f} < 0$ only if $w \in M$, and $t_{w,f} > 0$ only if $w \in m$.

Affirmative action policies aim to favor certain groups. A natural requirement is that a transfer should not reduce the welfare of minority workers. A transfer $\mathbf{T}$ *does not reduce the welfare of minority workers* under φ if, for a market $G = (\mathbf{T}_0)$ where $\varphi(\mathbf{T}_0) = (\mu, \mathbf{s})$ and $\varphi(\mathbf{T}) = (\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}})$, the following condition holds for each $w \in m$:

- $u_w(\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}}) \geq u_w(\mu, \mathbf{s})$.

A transfer $\mathbf{T}$ *reduces the welfare of minority workers* under φ if this condition is violated.[10] A transfer $\mathbf{T}$ *never reduces the welfare of minority workers* under φ if this condition holds for all markets $G \in \mathbb{G}$; otherwise, $\mathbf{T}$ *may reduce the welfare of minority workers*.

3 Uniform Transfers and Minority Workers Welfare

In this section, we examine the impact of commonly used targeted transfers. Using an example, we illustrate the potential adverse effects such transfers may have on minority workers.

Example 1. Consider a market consisting of three firms, $F = \{f_1, f_2, f_3\}$ and three workers, $W = \{w_1, w_2, w_3\}$. For simplicity, assume that for each worker $w \in W$ and each firm $f \in F$, $\sigma_{w,f} = 0$, also, for each $W' \subseteq W$,

$$R_f(W') = \begin{cases} R_f(\{w\}), & \text{if } W' = \{w\} \text{ for some } w \in W, \\ 0, & \text{otherwise.} \end{cases}$$

[7] The existence of these allocations in the context of job markets has been extensively studied by Kelso and Crawford (1982), Roth (1984), 1985, Demange and Gale (1985), Gul and Stacchetti (1999), Hatfield and Milgrom (2005) among others.

[8] A group of workers may be defined based on factors such as gender, ethnicity, qualifications, location, educational background, or any other combination of individual characteristics.

[9] Throughout the paper, we implicitly assume that minority (majority) workers are never taxed (subsidized).

[10] This can be seen as a weak Pareto improvement for minority workers. More specific objectives are discussed in Section 5.

Revenue functions are provided in Table 1,[11] while optimal stable allocations are detailed in Table 2.

Firms	w_1	w_2	w_3
f_1	5	4	0
f_2	1	5	0
f_3	5	0	3

Table 1: Revenue functions.

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 2$	$s_{w_2} = 1$	$s_{w_3} = 0$

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 5$	$s_{w_2} = 5$	$s_{w_3} = 3$

Table 2: Firm-optimal and worker-optimal stable allocations.

Let the partition of workers $\mathcal{P}$ be defined as $m = \{w_2, w_3\}$ and $M = \{w_1\}$. Suppose the policymaker's objective is to increase the representation of minority workers at firm f_1.[12] To achieve this objective, the policymaker introduces a subsidy of 2 for each minority worker at f_1; that is, $\mathbf{T}$ is defined by $t_{w_2,f_1} = 2$, $t_{w_3,f_1} = 2$, and zero for all other elements. The resulting worker-optimal stable allocation is given in Table 3.

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 4$	$s_{w_2} = 5$	$s_{w_3} = 2$

Table 3: Worker-optimal stable allocation with a subsidy of 2 at f_1.

The assignment is not modified, and w_3's salary decreases. The intuition is that, by subsidizing w_2 at firm f_1, firm f_1 is willing to offer a higher salary to w_2, which intensifies competition between w_1 and w_2. As a result, w_1 considers other firms, in particular f_3, which intensifies competition with worker w_3 and reduces w_3's salary.

Now suppose that the policymaker aims to increase the salary of w_3 at firm f_3 by granting a subsidy such that $t_{w_3,f_3} = 2$ and zero for all other elements. The resulting firm-optimal stable allocation is given in Table 4.

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 0$	$s_{w_2} = 0$	$s_{w_3} = 0$

Table 4: Firm-optimal stable allocation with a subsidy of 2 for worker w_3 at firm f_3.

[11] These revenue functions belong to a special class known as *unit-demand revenue functions*, which satisfy the gross substitutes condition.

[12] Such transfers may be implemented by policymakers to support specific sectors of the market with the aim of enhancing minority representation.

Before the introduction of the subsidy, firms f_1 and f_3 competed for worker w_1, which led to an increase in the salary of w_2, the worker generating the highest revenue for f_1 after w_1. Following the subsidy to w_3 at f_3, the firm becomes indifferent between employing w_1 and w_3. This reduces the competition between f_1 and f_3 for worker w_1, thereby lowering w_2's salary. Hence, directing subsidies to minority workers at specific firms can generate unintended adverse effects on other minority workers, both at the worker-optimal and firm-optimal stable allocations.

Another common instrument in affirmative action policies is the implementation of taxes. These taxes are designed to penalize firms that fail to meet government-imposed representation requirements. Firms may receive tax abatements for employing a minimum number of minority workers. In this context, imposing a tax is analogous to reducing the abatement. The key distinction between taxes and subsidies lies in the range of possible amounts. While there may be no mathematical distinction, from an economic perspective, it is important to limit the amount of the tax. If the tax exceeds the surplus generated by a worker at a firm, the firm will never employ that worker. Therefore, throughout this paper, it is assumed that, for each worker w and each firm f, $R_f(\{w\}) + \sigma_{w,f} + t_{w,f} \geq 0$.

We now illustrate the difference between taxes and subsidies. Consider the market presented in Table 1, and suppose a tax of one is imposed on worker w_1 at firm f_1, that is, $t_{w_1,f_1} = -1$, with all other elements equal to zero. The resulting optimal stable allocations are given in Table 5.

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 2$	$s_{w_2} = 2$	$s_{w_3} = 0$

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 4$	$s_{w_2} = 5$	$s_{w_3} = 2$

Table 5: Firm-optimal and worker-optimal stable allocations with a tax of 1 at f_1.

For the worker-optimal stable allocation, the effect mirrors that shown in Table 3: by imposing a tax on worker w_1 at firm f_1, worker w_1 aiming to maximize the salary competes for firm f_3, thereby intensifying the competition with worker w_3. In contrast, at the firm-optimal stable allocation, salaries are not reduced. Intuitively, under the worker-optimal stable allocation, workers compete to maximize their own salaries, whereas under the firm-optimal stable allocation, firms compete to maximize their profits. By taxing majority workers, minority workers become relatively more attractive, prompting firms to offer them higher salaries. This example shows that transfers may alter workers' welfare in an allocation, even if the transfers do not directly concern those workers. The following proposition formalizes that taxing majority workers can reduce the welfare of minority workers under φ_W, but never under φ_F.

Proposition 1. A transfer $\mathbf{T}$ that taxes majority workers may reduce the welfare of minority workers under φ_W and never reduces the welfare of minority workers under φ_F.

Proof. See Appendix D.1. ■

Although taxes do not reduce the welfare of minority workers under the firm-optimal stable allocation, they may do so under the worker-optimal stable allocation. Our first main theorem establishes

that uniform transfers form the entire class of transfers that never reduce the welfare of minority workers under φ_F and φ_W. A transfer $\mathbf{T}$ is *uniform* if there exist constants $t \geq 0$ and $t' \leq 0$ such that for each $w \in m$ and each $f \in F$, $t_{w,f} = t$, and for each $w' \in M$ and each $f \in F$, $t_{w',f} = t'$.

Theorem 1. A transfer $\mathbf{T}$ never reduces the welfare of minority workers under φ_F and φ_W if and only if $\mathbf{T}$ is uniform.

Proof. See Section D.3. ■

The policy implication of Theorem 1 is that if a transfer—be it a tax or a subsidy—is implemented to target a specific worker, subset of workers, firm, or sector, then it must be introduced uniformly across all minority workers and all firms in order to prevent a reduction in welfare. In addition, uniform transfers also have implications for implementation budgets. In particular, such transfers can be designed as a uniform tax, with the resulting revenue used to finance subsidies for all minority workers. As long as transfers are uniform, a budget-balanced policy can be implemented.
Moreover, uniform transfers are also the only transfers that prevent a decrease in the aggregate welfare of minority workers. A transfer $\mathbf{T}$ *has a positive aggregate effect on minority workers* under φ if, for a market $G = (\mathbf{T}_0)$ such that $\varphi(\mathbf{T}_0) = (\mu, \mathbf{s})$ and $\varphi(\mathbf{T}) = (\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}})$, we have $\sum_{w \in m}(u_w(\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}}) - u_w(\mu, \mathbf{s})) \geq 0$. A transfer $\mathbf{T}$ *always has a positive aggregate effect on minority workers* under φ if this condition holds for all markets $G \in \mathbb{G}$.

Corollary 1. A transfer $\mathbf{T}$ always has a positive aggregate effect on minority workers under φ_F and φ_W if and only if $\mathbf{T}$ is uniform.

Corollary 1 complements Theorem 1 by providing an alternative measure of welfare.[13] In addition, whereas any tax on majority workers prevents a reduction in minority welfare at the firm-optimal stable allocation, the following proposition shows that only uniform transfers prevent a decrease in minority welfare at the worker-optimal stable allocation.

Proposition 2. A transfer $\mathbf{T}$ never reduces the welfare of minority workers under φ_W if and only if $\mathbf{T}$ is uniform.

Proposition 2 complements Proposition 1 and follows from Theorem 1 by imposing the condition only on the worker-optimal stable mechanism.
These results highlight an important contrast with markets without transferable utility. In the school choice setting, Kojima (2012) shows that no affirmative action policy can be designed so as to avoid reducing minority welfare under any stable mechanism. In particular, he considers increasing students' priorities as a policy instrument. In the context of our model, increasing a minority worker's priority corresponds to providing a subsidy at a firm. This difference arises due to the continuity of salaries: in school choice, raising a student's priority modifies the ordinal ranking of schools; if the ranking is unchanged, the resulting assignment remains the same. By contrast, in markets with transferable utility, subsidizing a worker may not change the ordinal ranking of firms, yet still alter the allocation. We examine this distinction in greater detail in Appendix A.1.

[13] Sufficiency follows directly from Theorem 1, while necessity is established using the construction in the proof of Theorem 1; the proof is therefore omitted.

Example 1 (Continued). Consider a uniform transfer $\mathbf{T}$ that taxes the majority worker by 1. The resulting optimal stable allocations for this market are presented in Table 6.

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 1$	$s_{w_2} = 1$	$s_{w_3} = 0$

f_1	f_2	f_3
w_1	w_2	w_3
$s_{w_1} = 4$	$s_{w_2} = 5$	$s_{w_3} = 3$

Table 6: Firm-optimal and worker-optimal stable allocations with a uniform transfer.

Table 6 illustrates that uniform transfers prevent any reduction in the welfare of minority workers at both the firm-optimal and worker-optimal stable allocations. While these transfers may increase competition among workers, their uniform application across firms neutralizes potential negative effects by providing equal treatment to all minority workers. However, they do not guarantee any improvement. As shown in Table 6, the welfare of minority workers remains unchanged. Our next theorem establishes a sufficient condition on transfers that ensures a welfare gain for minority workers at the worker-optimal stable allocation.

Theorem 2. Consider a market $G \in \mathbb{G}$. If $\mathbf{T}$ subsidizes minority workers and satisfies $t_{w,\mu_W(w)} = \max_{f \in F}(t_{w,f})$ for each $w \in m$, then $\mu_W = \mu_W^{\mathbf{T}}$, and for each $w \in W$,

$$u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \geq u_w(\mu_W, \mathbf{s}_W) + t_{w,\mu_W(w)}.$$

Proof. See Appendix D.2. ■

Theorem 2 demonstrates that, given knowledge of the worker-optimal stable allocation, it is possible to design transfers that raise minority workers' salaries without altering the assignment. The key insight is that, because each minority worker receives the highest subsidy at their current employer, no worker has an incentive to consider employment at another firm. As a result, competition is not intensified, and the entire surplus generated by the subsidy is captured by the workers. Moreover, the welfare of all workers—both minority and majority—is weakly improved.[14] However, the transfers described in Theorem 2 may reduce the welfare of minority workers under φ_F, as shown in Table 4, where only w_3 receives a subsidy at f_3.

The design of transfer policies critically depends on the information available to the policymaker. When the mechanism implementing the allocation is unknown, uniform transfers are the only class of transfers that guarantee minority workers are not adversely affected under both the worker-optimal and firm-optimal stable allocations. If the policymaker observes that the market operates under the firm-optimal stable allocation, then any tax on majority workers—regardless of its structure—preserves the welfare of minority workers. However, such taxes may fail to improve minority outcomes. In contrast, if the equilibrium corresponds to the worker-optimal stable allocation, Theorem 2 provides a sufficient condition under which targeted subsidies raise minority salaries without reducing the welfare of majority workers. This result is particularly relevant in

[14] The increase in workers' utility may strictly exceed the amount of the transfer due to the potential reduction in competition resulting from the subsidy.

practice, as minority workers frequently earn lower salaries.[15] By allowing subsidies to vary across workers, this approach enables targeted interventions—such as sector-specific salary policies—that can improve welfare without changing the assignment.[16]

4 Affirmative Action with More than Two Groups

In this section, we relax the restrictions on the number of groups. In practice, populations may consist of more than two groups, with several of them classified as minority groups. An interpretation of Section 3 is that we considered the union of all minority groups and the union of all non-minority groups. While such aggregate treatment is analytically convenient, policymakers may seek to implement group-specific interventions. We therefore consider the same framework with a finer partition of workers, allowing transfers to differentiate across groups, and let $\mathcal{P}$ denote a partition of the worker population.

We extend the definitions for minority workers to any group P. A transfer $\mathbf{T}$ *does not reduce the welfare of group* P under φ if, for a market $G = (\mathbf{T}_0)$ where $\varphi(\mathbf{T}_0) = (\mu, \mathbf{s})$ and $\varphi(\mathbf{T}) = (\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}})$, the following condition holds for each $w \in P$:

- $u_w(\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}}) \geq u_w(\mu, \mathbf{s})$.

A transfer $\mathbf{T}$ *reduces the welfare of group* P under φ if this condition is violated. A transfer $\mathbf{T}$ *never reduces the welfare of group* P under φ if this condition holds for all markets $G \in \mathbb{G}$; otherwise, $\mathbf{T}$ *may reduce the welfare of group* P. A transfer $\mathbf{T}$ *uniformly subsidizes (taxes) group* P if there exists $t \geq 0$ ($t \leq 0$) such that, for each $w \in P$ and each $f \in F$, $t_{w,f} = t$, and for each $w' \in W \setminus P$, $t_{w',f} = 0$.

Our previous results highlight the importance of implementing uniform transfers to avoid a reduction in welfare. In particular, if a transfer does not subsidize each group uniformly and by the same amount, it may adversely affect some groups under both φ_W and φ_F.

Corollary 2. A transfer $\mathbf{T}$ that uniformly subsidizes group P may reduce the welfare of group P' under φ_W and φ_F, with $P' \in \mathcal{P} \setminus \{P\}$.

Theorem 1 and Corollary 2 demonstrate that all minority groups must be subsidized uniformly and by the same amount to avoid welfare losses. If one group receives larger subsidies than another, the latter may experience reductions in welfare.

The following corollary complements the characterization in Theorem 1 by showing that if a transfer uniformly taxes one group, it never reduces the welfare of any other group.

Corollary 3. A transfer $\mathbf{T}$ that uniformly taxes group P, never reduces the welfare of group P' under φ_W and φ_F, with $P' \in \mathcal{P} \setminus \{P\}$.

Corollary 3 establishes that, in the presence of multiple minority groups, uniform taxation of one group prevents welfare losses for the others. The argument follows from Theorem 1 by treating group

[15] The empirical literature documenting salary disparities faced by minority workers is too extensive to cite exhaustively. See, for instance, Derenoncourt and Montialoux (2021), who document these disparities and analyze the impact of minimum salary policies on minority workers' welfare.

[16] A sector-specific minimum salary can be interpreted as a direct application of Theorem 2.

P as the majority workers and workers in all remaining groups—that is, all groups in $\mathcal{P} \setminus \{P\}$—as the minority workers. Together, these results highlight the importance of transfer design when the worker population consists of several groups, including multiple minority groups.

5 Affirmative Action Policies

This section examines two common objectives of affirmative action policies: reducing the segregation of minority workers across firms or sectors, and reducing unemployment. We restrict attention to a setting with two groups—a majority and a minority.

Modifying the representation of minority workers in market allocations requires altering agents' incentives. In particular, it is necessary to determine the level of transfers required to compensate firms so as to make them indifferent between continuing to employ majority workers and replacing them with minority workers. To preserve tractability, we assume that workers are perfectly comparable from firms' perspectives and that firms are constrained in the number of workers they can employ, independently of workers' identities. Each firm $f \in F$ has a *capacity* $q_f \in \mathbb{N}$, and let $\mathbf{q} \equiv (q_f)_{f \in F}$ denote the *capacity vector*. To maintain competition among workers, we assume an excess supply of labor, $|W| > \sum_{f \in F} q_f$. The revenue function of firm f is *additively separable* subject to its capacity constraint; that is, for any subset of workers $W' \subset W$ such that $|W'| \leq q_f$, the revenue function satisfies $R_f(W') = \sum_{w \in W'} R_f(\{w\})$ and $R_f(W') = -\infty$ otherwise.[17] The null firm has capacity $q_\varnothing = |W|$, so all workers can remain unemployed. Since firms' revenue functions are additively separable, we simplify notation by writing $R_f(w)$ instead of $R_f(\{w\})$.

We define a market with capacities as $\overline{G} = (W, F, \boldsymbol{\sigma}, R, \mathbf{q}, \mathbf{T})$. Let $\overline{\mathbb{G}}$ be the class of such markets where all R_f are additively separable subject to capacity q_f. Since W, F, $\boldsymbol{\sigma}$, and $\mathbf{q}$ are fixed, and different markets correspond to variations in firms' revenue profiles R, we write $\overline{G} = (\mathbf{T})$. When revenue functions are additively separable and firms face capacity constraints, firm demand continues to satisfy the gross substitutes condition (Kojima et al. 2020). Consequently, markets with capacities constitute a subclass of the markets in $\mathbb{G}$, and the results from the previous sections extend to this environment.

Since there is excess supply of labor, some workers remain unemployed. Subsidizing these workers increases their attractiveness to firms, which can reduce the welfare of already employed workers without necessarily inducing the hiring of unemployed workers. The following example illustrates this effect.

Example 2. Consider a market with capacities consisting of two firms, $F = \{f_1, f_2\}$ and five workers, $W = \{w_1, w_2, w_3, w_4, w_5\}$. Each firm $f \in F$ has capacity $q_f = 2$, and for each worker $w \in W, \sigma_{w,f} = 0$. Table 7 presents the revenue functions, and Table 8 shows the firm-optimal and worker-optimal stable allocations.

Suppose that the partition of workers $\mathcal{P}$ is defined as $m = \{w_1, w_2, w_5\}$ and $M = \{w_3, w_4\}$, where w_5 is an unemployed minority worker. To reduce minority unemployment, the policymaker implements

[17] This assumption has been widely discussed in the literature; see, Crawford and Knoer (1981), Demange and Gale (1985), Sotomayor (1999), Kojima et al. (2024), among others. This assumption is naturally satisfied in many practical settings, such as unskilled or specialized labor markets.

Firms	w_1	w_2	w_3	w_4	w_5
f_1	8	7	5	4	2
f_2	3	6	8	5	3

Table 7: Revenue functions.

f_1		f_2	
w_1	w_2	w_3	w_4
$s_{w_1} = 0$	$s_{w_2} = 1$	$s_{w_3} = 0$	$s_{w_4} = 0$

f_1		f_2	
w_1	w_2	w_3	w_4
$s_{w_1} = 6$	$s_{w_2} = 5$	$s_{w_3} = 5$	$s_{w_4} = 2$

Table 8: Firm-optimal and worker-optimal stable allocations.

a subsidy: $t_{w_5,f_1} = t_{w_5,f_2} = 1$, with all other elements of $\mathbf{T}$ equal to zero.[18] The resulting optimal stable allocations are shown in Table 9.

f_1		f_2	
w_1	w_2	w_3	w_4
$s_{w_1} = 0$	$s_{w_2} = 1$	$s_{w_3} = 0$	$s_{w_4} = 0$

f_1		f_2	
w_1	w_2	w_3	w_4
$s_{w_1} = 5$	$s_{w_2} = 4$	$s_{w_3} = 4$	$s_{w_4} = 1$

Table 9: Firm-optimal and worker-optimal stable allocations.

Table 9 illustrates that subsidizing unemployed minority workers can intensify market competition, thereby reducing the welfare of employed minority workers. To design transfers that do not reduce the welfare of minority workers, we distinguish between employed and unemployed workers within the minority group.

Given a market with capacities $\overline{G} \in \overline{\mathbb{G}}$ and a worker-optimal stable allocation $(\mu_W, \mathbf{s}_W)$, let

$$m_E \equiv \{w \in m : \mu_W(w) \neq \varnothing\} \quad \text{and} \quad m_U \equiv \{w \in m : \mu_W(w) = \varnothing\},$$

denote the sets of *employed* and *unemployed* minority workers, respectively. We refer to the assignment from the worker-optimal stable allocation. Note that, generically, all stable allocations share the same assignment and differ only in the associated vector of salaries; we impose this assumption throughout this section.[19]

When a transfer $\mathbf{T}$ is introduced, we define

$$m_E^{\mathbf{T}} \equiv \{w \in m : \mu_W^{\mathbf{T}}(w) \neq \varnothing\} \quad \text{and} \quad m_U^{\mathbf{T}} \equiv \{w \in m : \mu_W^{\mathbf{T}}(w) = \varnothing\},$$

as the sets of employed and unemployed minority workers under the resulting stable allocation.

The following proposition provides a sufficient condition on transfers that guarantees no reduction

[18] Such targeted subsidies are commonly used to support the employment of disadvantaged groups.

[19] The generic uniqueness assumption is standard in the literature (see, for example, Shapley and Shubik (1971)). When firms face capacity constraints and revenue functions are additively separable, Sotomayor (1999) shows that the assignment is unique across stable allocations. Markets with multiple assignments arise when some workers generate the same revenue for certain firms. In such cases, assignments differ only through permutations of these workers, while all other matches remain identical.

in the welfare of minority workers under the worker-optimal stable allocation.

Proposition 3. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Suppose a transfer $\mathbf{T}$ subsidizes minority workers and satisfies, for each $w \in m_E$,

- $t_{w,\mu_W(w)} = \max_{f \in F}(t_{w,f})$, and
- $t_{w,\mu_W(w)} \geq \max_{w' \in m_U, f' \in F}(t_{w',f'})$.

Then, $\mathbf{T}$ does not reduce the welfare of minority workers under φ_W.

Proof. See Section D.5. ■

Proposition 3 states that when subsidies are granted to unemployed workers, employed minority workers must receive a compensation to prevent a reduction in welfare under the worker-optimal stable allocation. As illustrated by Example 2, providing subsidies to unemployed workers increases their desirability for firms, which can lead firms to offer lower salaries or reassign positions, thereby reducing the welfare of employed minority workers. Granting the corresponding compensation to the firm employing a given worker ensures that the worker's utility is preserved, as formalized in Theorem 2.

The policy implication of Proposition 3 is that subsidies targeting unemployed minority workers must be complemented by subsidies of at least the same magnitude for employed minority workers. In practice, however, many affirmative action policies are directed exclusively at the unemployed, without accounting for potential welfare losses among employed workers.

5.1 Increasing the Representation of Minority Workers in a Firm

In this section, we explore transfers that increase the representation of minority workers in a firm. A transfer $\mathbf{T}$ *increases the representation of minority workers in firm* f under φ if, given a market with capacities $\overline{G} = (\mathbf{T}_0)$ where $\varphi(\mathbf{T}_0) = (\mu, \mathbf{s})$ and $\varphi(\mathbf{T}) = (\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}})$, the following two conditions hold:

(i) $q_f \geq |\mu^{\mathbf{T}}(f) \cap m| > |\mu(f) \cap m|$, and
(ii) $(\mu(f) \cap m) \subset (\mu^{\mathbf{T}}(f) \cap m)$.

Condition (i) ensures that at least one additional minority worker is hired by firm f.[20] Condition (ii) guarantees that no previously employed minority workers lose their job. Together, these imply that the increased representation of minority workers necessarily comes at the expense of majority workers employed by f prior to the transfer.

For simplicity and without loss of generality, we consider a single minority worker targeted for employment at a firm, displacing a majority worker. The displaced majority worker is the one who contributes the least profit to the firm under the firm-optimal stable allocation.[21] Let w_m and w_M

[20] The reference to firm capacity in condition (i) ensures that firm f must initially employ majority workers who can be replaced.

[21] Under the worker-optimal stable allocation, all workers employed at a given firm generate the same profit (see Lemma 4). We therefore rely on the firm-optimal stable allocation to differentiate among workers. If multiple majority workers generate the same lowest profit at f, the reasoning remains valid: any one of these workers can be substituted by w_m in a stable allocation without affecting the results.

denote the minority and majority workers, respectively, with $\mu_F(w_m) \neq f$ and $\mu_F(w_M) = f$, and let

$$w_M \in \arg \min_{w \in \mu_F(f) \cap M} (R_f(w) - s_{Fw,f}),$$

be a majority worker contributing the least profit to firm f. For w_m to be hired by firm f instead of w_M, the minority worker w_m must generate a higher profit at f than w_M. In addition, the firm currently employing w_m may have alternative hiring options, which we assume to be the set of unemployed workers $W_U \equiv \{w \in W : \mu_W(w) = \varnothing\}$. We define $t_{w_m \to f}$ as the *subsidy required for* w_m *to be hired by firm* f. It is given by:

$$t_{w_m \to f} \equiv \overbrace{(R_f(w_M) - s_{Fw_M,f}) - (R_f(w_m) + \sigma_{w_m,f} - u_{w_m}(\mu_F, \mathbf{s}_F))}^{\text{Compensation to firm } f \text{ for the substitution of } w_M \text{ by } w_m} + \underbrace{R_{\mu_F(w_m)}(w_m) + \sigma_{w_m,\mu_F(w_m)} - u_{w_m}(\mu_F, \mathbf{s}_F) - \max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} (R_{\mu_F(w_m)}(w') + \sigma_{w',\mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F))}_{\text{Compensation to the current employer of } w_m}.$$

The required subsidy depends on the revenue functions of both firm f and $\mu_F(w_m)$.[22] The first term ensures that firm f is willing to substitute w_M with w_m by covering the difference in profits. The second term compensates w_m's current employer, ensuring it cannot outbid firm f by offering a higher salary.[23] If w_m is unemployed, this second term is zero, as no compensation is needed to retain the worker in the current position.

Example 2 (Continued). Suppose the set of minority workers is given by $m = \{w_1, w_4\}$, and that the policymaker aims to increase the representation of minority workers at firm f_1 by ensuring the employment of w_4. The required subsidy is $t_{w_4 \to f_1} = 2$. Let $\mathbf{T}$ be a transfer such that $t_{w_4,f_1} = 2$, with all other elements of $\mathbf{T}$ equal to zero. The resulting optimal stable allocations are presented in Table 10.

f_1		f_2	
w_1	w_4	w_2	w_3
$s_{w_1} = 0$	$s_{w_4} = 0$	$s_{w_2} = 1$	$s_{w_3} = 0$

f_1		f_2	
w_1	w_4	w_2	w_3
$s_{w_1} = 4$	$s_{w_4} = 2$	$s_{w_2} = 3$	$s_{w_3} = 5$

Table 10: Firm-optimal worker-optimal stable allocations with a subsidy of 2 for w_4 at f_1.

Although w_4 is now employed by firm f_1, the salary of the minority worker w_1 has decreased. This example illustrates that such a transfer can reduce the welfare of minority workers already employed at firm f under the worker-optimal stable allocation.

The following theorem provides a sufficient condition on transfers for a given market with capacities to increase the representation of minority workers in a firm and to prevent a reduction in the welfare of minority workers at the worker-optimal stable allocation.

[22] Claim 3 shows that $t_{w_m \to f}$ enables w_m to generate (weakly) higher profit at firm f than w_M at the firm-optimal stable allocation.

[23] We assume that the best alternative for the firm $\mu_F(w_m)$ is either an unemployed worker or w_M. There may exist an employed worker who constitutes a better alternative; however, this does not affect our results, since $t_{w_m \to f}$ is larger when defined with respect to the unemployed worker than with the best employed alternative.

Theorem 3. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Suppose a transfer $\mathbf{T}$ subsidizes minority workers and satisfies the following conditions:

- For each $w \in m_E \setminus \{w_m\}$ such that $\mu_W(w) \neq f$, $t_{w,\mu_W(w)} = \max_{f' \in F}(t_{w,f'}) \geq t_{w_m \to f}$,
- For each $w \in m_E$ such that $\mu_W(w) = f$, $t_{w,f} \geq \max_{f' \in F \setminus \{f\}}(t_{w,f'}) + t_{w_m \to f}$,
- For each $w' \in W \setminus (m_E \cup \{w_m\})$ and each $f' \in F$, $t_{w',f'} = 0$, and
- For each $f' \in F \setminus \{f\}$, $t_{w_m,f'} = 0$, and $t_{w_m,f} \geq t_{w_m \to f}$.

Then, $\mathbf{T}$ does not reduce the welfare of minority workers under φ_W and increases the representation of minority workers in firm f under any stable mechanism.

Proof. See Section D.6. ■

In contrast to the salary increases discussed in Theorem 2, modifying the assignment of minority workers requires a non-uniform transfer targeted specifically at the minority workers employed by firm f. Theorem 3 establishes that these workers must receive a subsidy at firm f that is at least $t_{w_m \to f}$ higher than any subsidy they would receive at other firms. This condition ensures that the workers remain employed at f and that the transfer does not reduce their welfare under φ_W.

For minority workers who are employed but not by f, their highest subsidy must be provided by their current employer, and it must be at least $t_{w_m \to f}$, as presented in Proposition 3. While w_m is chosen arbitrarily, it is possible that other minority workers could, in principle, be hired by firm f with lower subsidies. However, by the construction of $\mathbf{T}$, even if such workers exist, they remain employed at their original firm. In the proof of Theorem 3, we apply the same argument as in Theorem 2, showing that the assignment remains unchanged. Consequently, only w_m is newly recruited by firm f.

The proof of Theorem 3 proceeds by decomposing the transfer $\mathbf{T}$ into a sum of transfers (Lemma 1, Appendix C). The first transfer subsidizes all employed minority workers other than w_m, according to the amounts they receive under $\mathbf{T}$. By Theorem 2, this transfer increases the salaries of those workers. We then introduce a transfer that subsidizes w_m by $t_{w_m \to f}$. Lemma 2 (Appendix C) implies that, as a consequence of this transfer, the utility of any worker can decrease by at most $t_{w_m \to f}$. Since their previous utility gains exceeded $t_{w_m \to f}$, the welfare of minority workers is not reduced. The remaining subsidy for w_m at f is then introduced, allowing w_m to be recruited by f and being subsidized by $t_{w_m,f} - t_{w_m \to f}$. This, following Theorem 2, results in an increase in w_m's salary by at least that amount, without reducing the salaries of other workers. However, as in Theorem 2, the transfers considered in Theorem 3 may reduce the welfare of minority workers under φ_F.[24]

Proposition 4 complements Theorem 3 by identifying a sufficient condition for increasing the representation of minority workers in firm f, without imposing any restrictions on the welfare of minority workers. Specifically, only the positions of the minority workers employed at firm f must be maintained, and some minority workers may experience a reduction in their welfare.

[24] In Example 2, let $m = \{w_2, w_5\}$, and suppose the objective is to ensure that w_5 is employed by firm f_2. Then $t_{w_5 \to f_2} = 2$. Consider a transfer $\mathbf{T}$ with $t_{w_2,f_1} = 2$, $t_{w_5,f_2} = 3$, and 0 for other elements, which satisfies the conditions identified in Theorem 3. Under $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$, the salary of w_2 is 0.

Proposition 4. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Suppose a transfer $\mathbf{T}$ subsidizes minority workers and satisfies the following conditions:

- For each $w \in m_E$ such that $\mu_W(w) = f$, $t_{w,f} \geq \max(0, R_f(w_M) - s_{Fw_M,f} - (R_f(w) - s_{Fw,f}))$,
- For each $w' \in W \setminus (m_E \cap (\mu_F(f) \cup \{w_m\}))$, and each $f' \in F$, $t_{w',f'} = 0$, and
- For each $f' \in F \setminus \{f\}$, $t_{w_m,f'} = 0$, and $t_{w_m,f} \geq t_{w_m \to f}$.

Then, $\mathbf{T}$ increases the representation of minority workers in firm f under any stable mechanism.

Proof. See Section D.7. ■

For their positions at firm f to be maintained, minority workers already employed by f must generate a profit higher than that of w_M. The class of transfers identified in Proposition 4 achieves this by subsidizing each worker by at least 0 if the profit they generate at f already exceeds that generated by w_M, and by the difference otherwise.

5.2 Reducing Unemployment Among Minority Workers

In this section, we consider a policy aimed at reducing unemployment among minority workers. A transfer $\mathbf{T}$ *promotes the employment of minority workers* under φ if, for a market with capacities $\overline{G} = (\mathbf{T}_0)$, where $\varphi(\mathbf{T}_0) = (\mu, \mathbf{s})$ and $\varphi(\mathbf{T}) = (\mu^{\mathbf{T}}, \mathbf{s}^{\mathbf{T}})$, the following two conditions hold:

(i) $|m_E^{\mathbf{T}}| > |m_E|$, and

(ii) $m_E \subset m_E^{\mathbf{T}}$.

Condition (i) ensures that at least one additional minority worker is employed, while condition (ii) guarantees that currently employed minority workers retain their employment. Together, these conditions imply that the increase in minority employment occurs at the expense of majority workers. In defining the objectives in this section, two distinctions arise between promoting the employment of minority workers and increasing their representation at a specific firm. First, under the former, minority workers who are already employed may be reassigned to another firm, provided they remain employed after the transfer, whereas under the latter they must remain employed by the same firm. Second, the former requires that unemployed minority workers obtain employment with some firm, without specifying which one, while the latter requires an increase in representation at a given firm. Hence, the two objectives are independent.

As in the previous section, and without loss of generality, we consider a single unemployed minority worker displacing a majority worker at a firm. Let $w_m \in m_U$ denote the targeted minority worker. The firm f^* is chosen to minimize the required subsidy to implement this substitution. Specifically,

$$f^* \in \arg\min_{f \in F} \left(\min_{w \in \mu_F(f) \cap M} (R_f(w) + \sigma_{w,f} - R_f(w_m) - \sigma_{w_m,f}) \right).$$

Let w_M be a majority worker at firm f^* who generates the lowest surplus at that firm, that is, $w_M \in \arg\min_{w \in \mu_F(f^*) \cap M} (R_{f^*}(w) + \sigma_{w,f^*})$.

We define $t_{w_m \to m_E}$ as the *subsidy required for worker w_m to be hired by firm f^**,[25] given by:

$$t_{w_m \to m_E} \equiv R_{f^*}(w_M) + \sigma_{w_M,f^*} - (R_{f^*}(w_m) + \sigma_{w_m,f^*}).$$

[25] We prove that $t_{w_m \to m_E}$ is sufficient to guarantee that w_m is employed in the proof of Proposition 5.

As discussed in the previous section, since w_m is unemployed, no firm is concerned with maintaining their position by offering a higher salary. Therefore, only the firm f^* must be compensated to implement the substitution of w_m for w_M.

The following theorem identifies a sufficient condition on transfers, in a market with capacities, that ensures both the promotion of minority worker employment and the preservation of minority worker welfare at the worker-optimal stable allocation.

Theorem 4. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Suppose a transfer $\mathbf{T}$ subsidizes minority workers and satisfies the following conditions:

- For each $w \in m_E$, $t_{w,\mu_W(w)} = \max_{f \in F}(t_{w,f}) \geq t_{w_m \to m_E}$,
- For each $w' \in W \setminus (m_E \cup \{w_m\})$ and each $f \in F$, $t_{w',f} = 0$, and
- For each $f \in F \setminus \{f^*\}$, $t_{w_m,f} = 0$, and $t_{w_m,f^*} \geq t_{w_m \to m_E}$.

Then, $\mathbf{T}$ does not reduce the welfare of minority workers under φ_W and promotes the employment of minority workers under any stable mechanism.

Proof. See Section D.9. ■

Theorem 4 establishes that transfers in which each minority worker receives the highest subsidy at the firm that employs them, exceeding $t_{w_m \to m_E}$, while w_m receives a subsidy of at least $t_{w_m \to m_E}$ at f^*, and all other workers receive no subsidy, do not reduce the welfare of minority workers under φ_W. Moreover, these transfers promote the employment of minority workers under any stable mechanism.[26]

For employed minority workers, the requirement that the largest subsidy be granted by their current employer follows directly from Theorem 2. The proof proceeds analogously to that of Theorem 3, decomposing the transfer. In particular, the salaries of employed minority workers decrease by at most $t_{w_m \to m_E}$; by providing a higher subsidy, the transfer ensures that their welfare is not reduced.

While the transfers identified in Theorem 3 are non-uniform—subsidizing employed minority workers at firm f differently—some transfers identified in Theorem 4 are uniform. This distinction highlights that a uniform transfer, which subsidizes minority workers, and is at least $t_{w_m \to m_E}$, does not reduce the welfare of minority workers and promotes the employment of minority workers under any stable mechanism.

However, if the subsidy equals exactly $t_{w_m \to m_E}$, the utility of the minority worker w_m is zero in all stable allocations when the transfer is implemented. As shown in Theorem 2, once employed, w_m's salary can be increased without reducing the welfare of other workers. If the subsidy strictly exceeds $t_{w_m \to m_E}$, then, in addition to promoting the employment of w_m, the worker's salary can be increased, thereby guaranteeing a minimum salary.

The policymaker may aim to reduce unemployment without imposing restrictions on the welfare of employed minority workers. Proposition 5 provides a sufficient condition on transfers to maintain the employment of minority workers while hiring at least one previously unemployed minority worker.

[26] By contrast with Theorem 3, minority workers already employed at f^* are not subject to special treatment relative to minority workers employed at other firms. This distinction arises because the employment of minority workers must be preserved, but not necessarily at the same firm.

Proposition 5. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Suppose a transfer $\mathbf{T}$ subsidizes minority workers and satisfies the following conditions:

- For each $w \in m_E$, $t_{w,\mu_W(w)} = \max_{f \in F}(t_{w,f}) \geq \max(0, t_{w_m \to m_E} - (s_{Ww,\mu_W(w)} + \sigma_{w,\mu_W(w)}))$,
- For each $w' \in W \setminus (m_E \cup \{w_m\})$ and each $f \in F$, $t_{w',f} = 0$, and
- For each $f \in F \setminus \{f^*\}$, $t_{w_m,f} = 0$, and $t_{w_m,f^*} \geq t_{w_m \to m_E}$.

Then, $\mathbf{T}$ promotes the employment of minority workers under any stable mechanism.

Proof. See Section D.8. ■

Similar to Proposition 4, the utility of minority workers must remain positive after the introduction of the transfer to ensure that they are employed by a firm.[27]

5.3 Implementation of Objectives, Costs, and Surplus Sharing

The previous sections show that promoting the employment of minority workers and increasing their representation can be achieved through appropriately designed transfers. However, such policies may come with a significant cost. This section demonstrates that these costs can be mitigated when the objective is to reduce unemployment.

Interestingly, Theorem 5 establishes the substitution between subsidies and taxes in uniform transfers. It shows that introducing a subsidy of t through a uniform transfer—without taxing majority workers—yields an allocation equivalent to one in which majority workers are taxed by t, while minority workers receive no subsidy, provided that the magnitude of t preserves positive profits for firms.

Theorem 5. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and two uniform transfers, $\mathbf{T}$ and $\mathbf{T}'$, where $\mathbf{T}$ subsidizes minority workers by t, and $\mathbf{T}'$ taxes majority workers by t'. Suppose that, for each $f \in F$ and $w \in M$,

$$R_f(w) + t'_{w,f} + \sigma_{w,f} - u_w(\mu_W^{\mathbf{T}'}, \mathbf{s}_W^{\mathbf{T}'}) > 0. \tag{1}$$

If $t = |t'|$, then, there exist worker-optimal and firm-optimal stable allocations such that

$$(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) = (\mu_W^{\mathbf{T}'}, \mathbf{s}_W^{\mathbf{T}'}) \text{ and } (\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) = (\mu_F^{\mathbf{T}'}, \mathbf{s}_F^{\mathbf{T}'}).$$

Proof. See Section D.4. ■

Condition (1) requires that, after receiving a compensation that preserves their initial utility levels, all majority workers continue to generate positive profits for every firm. Economically, this condition ensures that the tax is not excessively high. It therefore bounds the magnitude of both the subsidy and the tax, while guaranteeing that the allocation induced by transfer $\mathbf{T}$ remains individually rational for firms in the market with capacities under transfer $\mathbf{T}'$. Given this equivalence, taxes and

[27] Sotomayor (1999) shows that multiple worker-optimal and firm-optimal stable allocations can exist in a given market. However, worker utility remains identical across all worker-optimal stable allocations, and similarly for all firm-optimal stable allocations (Theorem 1 of Sotomayor 1999). Due to the construction of the transfers presented in Proposition 5, some minority workers will have a utility of 0 in the worker-optimal stable allocation. This implies that in some worker-optimal stable allocations, these workers will not be employed. To guarantee that these workers are employed in all worker-optimal stable allocations, it is sufficient to consider strict inequalities.

subsidies are substitutable in uniform transfers when firm revenue functions are additively separable. Corollary 4 complements Theorem 5 by demonstrating that, under uniform transfers, combining taxes and subsidies yields additive effects on optimal stable allocations.

Corollary 4. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and three uniform transfers $\mathbf{T}, \mathbf{T}', \mathbf{T}''$ with $t \geq 0$, $t' \leq 0$, such that:

- $\mathbf{T}$ subsidizes minority workers by t and taxes majority workers by t'.
- $\mathbf{T}'$ subsidizes minority workers by $t + |t'|$.
- $\mathbf{T}''$ taxes majority workers by $t + |t'|$.

Suppose that, for each $f \in F$ and $w \in M$,

$$R_f(w) + t''_{w,f} + \sigma_{w,f} - u_w(\mu_W^{\mathbf{T}''}, \mathbf{s}_W^{\mathbf{T}''}) > 0.$$

Then, there exist worker-optimal and firm-optimal stable allocations such that:

$$(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) = (\mu_W^{\mathbf{T}'}, \mathbf{s}_W^{\mathbf{T}'}) = (\mu_W^{\mathbf{T}''}, \mathbf{s}_W^{\mathbf{T}''}) \text{ and } (\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) = (\mu_F^{\mathbf{T}'}, \mathbf{s}_F^{\mathbf{T}'}) = (\mu_F^{\mathbf{T}''}, \mathbf{s}_F^{\mathbf{T}''}).$$

Theorem 5 and Corollary 4 yield direct insights for the design and cost of affirmative action policies. To reduce policy costs, policymakers can prioritize taxation over subsidies within uniform transfers. For instance, a uniform transfer that imposes a tax of $t_{w_m \to m_E}$ on majority workers promotes the employment of minority workers. Moreover, the resulting tax revenue can be used to subsidize minority workers, for example, to guarantee a minimum salary. Taxes alone, however, do not guarantee higher salaries for minority workers, as taxing majority workers increases the relative attractiveness of all minority workers, thereby intensifying competition within the group.

Our next result establishes that when subsidies are used to reduce unemployment or to increase the representation of minority workers within firms, these transfers generate additional surplus, part of which is retained by the firms.

Proposition 6. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Suppose a transfer $\mathbf{T}$ subsidizes minority workers and, for some $f \in F$, $|\mu_W^{\mathbf{T}}(f) \cap m| > |\mu_W(f) \cap m|$. Then, there exists $w \in \mu_W^{\mathbf{T}}(f)$ such that,

$$t_{w,f} > u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) - u_w(\mu_W, \mathbf{s}_W),$$

and at least one of the following holds:

- $V_f(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_f, \mathbf{T}) > V_f(\mu_W; \mathbf{s}_W, R_f, \mathbf{T}_0)$, or
- $V_{\mu_W(w)}(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_{\mu_W(w)}, \mathbf{T}) > V_{\mu_W(w)}(\mu_W; \mathbf{s}_W, R_{\mu_W(w)}, \mathbf{T}_0)$.

Proof. See Section D.10. ■

This result highlights that changes in firm composition induced by subsidies increase the profits of some firms. This observation holds for both policies. In contrast, in the transfers described in Theorem 2, the surplus generated by the transfer is fully absorbed by minority workers, and firm profits are not increased. In the proof of Proposition 6, we show that if $t_{w,f} = 0$, then at least the firm $\mu_W(w)$ experiences an increase in profit; otherwise, if $t_{w,f} > 0$, it is firm f that benefits.

6 Discussion and Conclusion

In this section, we illustrate the applications of our results. Although affirmative action policies are implemented in many countries, we consider two well-known policies—one in the United States and one in Germany—as representative examples. Our analysis applies directly to these policies. We then discuss the role of lump-sum taxes in the labor market and the insights provided by our study.

6.1 United States: *Work Opportunity Tax Credit*

The *Work Opportunity Tax Credit* (WOTC) was introduced in the United States in 1996 to incentivize firms to hire workers from disadvantaged backgrounds and was in place until 2025. The program primarily targeted African Americans and Latinos from low-income households, former inmates, veterans, youth from impoverished neighborhoods, and individuals with disabilities. Firms participating in the program could receive a tax credit of up to $9,600 per eligible employee. In 2023, a total of 1,988,528 workers benefited from the program, amounting to $4.7 billion in tax credits—an average of approximately $2,400 per worker.[28] Notably, the tax credit varied across groups and depended both on individual and firm characteristics. In our framework, we interpret the WOTC as a non-uniform subsidy.

Our results show that when subsidies vary across individuals and across firms (e.g., by sector), they may inadvertently reduce the welfare of the very groups they are intended to support—under both the firm-optimal and worker-optimal stable allocations. Moreover, differentiating subsidies across multiple minority groups may reduce their welfare (Corollary 2). To avoid this unintended consequence while still promoting minority employment, we identify two policy options:

(i) *Implementing a uniform subsidy* for all targeted workers across all firms (Theorem 1), or

(ii) *Imposing a uniform tax* on all other workers across all firms (Theorem 1 and Corollary 3).

If the magnitude of the uniform transfer is sufficiently large, these policies not only avoid reducing minority welfare but may also reduce their unemployment (Theorem 4).

6.2 Germany: *Eingliederungszuschuss*

The *Eingliederungszuschuss* was introduced as part of Germany's 2002 labor market reform, commonly known as *Hartz IV*. This program aims to facilitate the integration of individuals facing barriers to labor market participation, including the long-term unemployed, persons with disabilities, older workers, and those from ethnic minorities. It incentivizes firms to hire by subsidizing up to 50% of the worker's salary for a period ranging from 12 to 36 months. The amount and duration of the subsidy depend on the worker's age and disability status. In 2024, 1,029,000 workers benefited from this subsidy, with a total expenditure of €28.7 billion—an average of approximately €27,891 per worker.[29] Although the program is available to all sectors, subsidies are concentrated

[28] WOTC statistics for 2023 https://www.cmswotc.com/work-opportunity-tax-credit-statistics-2023/ (Accessed on 05/05/2026).

[29] Eingliederungszuschuss statistics for 2024 https://www.destatis.de/DE/Themen/Gesellschaft-Umwelt/Soziales/Sozialhilfe/eingliederungshilfe.html (Accessed on 05/05/2026).

in industrial, service, and retail sectors, particularly among firms employing low-skilled labor.[30]
As with the WOTC, subsidies under the Eingliederungszuschuss vary by worker and firm characteristics. Moreover, the program is targeted exclusively at unemployed workers. Our results suggest that this design may reduce the welfare of employed minority workers. To prevent such unintended effects, policy should extend subsidies to employed minority workers as well, with transfers at least as large as those offered to unemployed workers (Proposition 3). A further goal of the program is to increase minority representation in specific sectors. As shown in Theorem 3, this cannot be achieved with uniform transfers. When targeting specific sectors, our results show that employed minority workers must receive a sector-specific subsidy to ensure no welfare loss and to support representation goals.

6.3 Lump-Sum Taxation

In our framework, taxes are levied directly on firms. This contrasts with the model of Dupuy et al. (2020a), in which taxation applies to transfers between agents—that is, to salaries. More specifically, they analyze how taxation affects the *value* of a market allocation, that is, the sum of workers' utilities and firms' profits.[31] Under such taxation, transfers between firms and workers become imperfect, as part of the surplus is captured by a central authority. As in their model, we do not explicitly represent this authority.
Uniform transfers can be interpreted as *lump-sum taxes*: rather than taxing a fixed proportion of the worker's transfer, a fixed amount is deducted per worker. Lump-sum taxes are common in labor markets and may take the form of hiring costs (e.g., employee healthcare contributions) or employment entry costs (e.g., licensing requirements or mandatory training). Theorem 2 of Dupuy et al. (2020a),[32] shows that reducing a lump-sum tax on transfers increases the value of the market allocation. Our analysis complements theirs in two key respects.
First, we study how uniform transfers affect the distribution of welfare across workers. Uniform subsidies, for instance, capture exemptions from hiring costs or other entry barriers to employment. Second, in markets with capacities where firms have additively separable revenue functions, we show that a uniform transfer—whether implemented as a tax on majority workers or as a subsidy to minority workers—of equal magnitude yields the same allocation. In this case, only firms' profits are affected, clarifying how reductions in total value are distributed among market participants.

6.4 Concluding Remarks

This paper examines the impact of transfer policies on minority workers in the labor market. We show that transfers targeting specific segments of the population can reduce the well-being of minority workers. In practice, such targeted interventions are widely used in affirmative action policies,

[30] A similar initiative, known as the *emplois francs* program, was introduced in France in 2018, aimed at supporting workers from priority urban areas. It provides a financial subsidy of up to €5,000 per worker each year, with a maximum duration of three years.

[31] It is well known that any competitive equilibrium yields the assignment that maximizes total market value (Shapley and Shubik 1971). Differences across equilibria arise only in the division of surplus among agents.

[32] Theorem 2 is presented in Appendix D in the working paper version (Dupuy et al. 2020b). I am grateful to Scott Duke Kominers for highlighting how this result fits into the broader framework developed in the present paper.

highlighting the importance of careful policy design. A primary concern is that minority workers should not experience a reduction in welfare when a policy is introduced. We show that only uniform transfers satisfy this requirement in all markets. Moreover, this result is robust to the choice of welfare measure, as it also holds when considering aggregate changes in utility.

When the policymaker has more detailed information about the market and its allocation, we identify transfers that achieve common objectives such as reducing unemployment, increasing minority representation, and raising salaries. These results have direct policy implications for labor markets in many countries, by providing guidance on the design of interventions aimed at altering worker representation. In particular, to change the distribution of workers across firms, we show that transfers can be used as an alternative to quotas or reservation policies. Moreover, such transfers can improve representation without reducing the welfare of other minority workers.

Our approach focuses on the firm-optimal and worker-optimal stable allocations. Since the set of stable allocations forms a lattice in our framework, these two extreme allocations provide natural bounds: workers attain their lowest utility at the firm-optimal stable allocation and their highest utility at the worker-optimal stable allocation. By focusing on these two allocations, we characterize the range of possible welfare outcomes for workers. A key implication of uniform transfers is therefore that minority welfare is not reduced at either extreme of this interval.

The analysis extends naturally to settings of object allocation. In many applications, prices are agent-specific (Gul et al. 2019); in this context, uniform transfers correspond to price adjustments based on group identity. Applications such as electricity markets or telecommunications auctions may thus benefit from group-specific subsidies or taxes. These results illustrate how transfers can be used to favor particular groups of agents.

A Additional Insights

In this appendix, we provide further insights on questions that may arise from our main analysis. First, we examine the distinction between transferable utility (TU) and non-transferable utility (NTU) settings. Second, we explore whether the results from Section 5.3 extend to other revenue functions within the class of demand satisfying the gross substitutes condition.

A.1 Distinction Between Markets With and Without Transfers

In NTU markets, Kojima (2012) demonstrates the impossibility of designing affirmative action policies that never reduce the welfare of minority agents within the class of stable mechanisms. In this section, we highlight a key distinction between NTU and TU markets, using the terminology of agents and institutions.

Example 3. Consider a market consisting of two institutions, i_1 and i_2, and three agents, a_1, a_2, and a_3. Each institution has one available position. Agents are evaluated with institution-specific scores as follows:[33]

[33] These scores may represent entrance exam results in a college admissions context or revenue functions in a labor market setting.

	a_1	a_2	a_3
i_1	6	3	2
i_2	6	7	5

Table 11: Scores of individuals across institutions.

We first consider the NTU market. To do so, we specify the preferences of agents and institutions, which correspond to ordinal rankings of institutions and agents, respectively. Institutions rank agents according to the scores reported in Table 11. Agents' preferences are given by:

- a_1: $i_2 \succ i_1$, $\quad a_2$: $i_1 \succ i_2$, $\quad a_3$: $i_2 \succ i_1$

In the agent-optimal stable assignment, a_2 is assigned to i_1 and a_1 to i_2. In the institution-optimal stable assignment, a_1 is assigned to i_1 and a_2 to i_2.

Now suppose that a_2 and a_3 belong to a disadvantaged group and receive a bonus of $+2$ to their scores at every institution. The updated scores are:

	a_1	a_2	a_3
i_1	6	5	4
i_2	6	9	7

Table 12: Scores of agents after applying the bonus across institutions.

At institution i_1, the ordinal ranking of agents remains unchanged, while at institution i_2 it is altered. As a result, the agent-optimal and institution-optimal stable assignments now coincide: a_1 is assigned to i_1 and a_2 to i_2. Agent a_2, who was initially assigned to i_1, is now worse off under the agent-optimal stable assignment.

We now consider the TU version of this market with capacities (Section 5). We normalize $\sigma_{a,i} = 0$ for each agent $a \in \{a_1, a_2, a_3\}$ and insitution $i \in \{i_1, i_2\}$.

Prior to the score adjustment, in the agent-optimal stable allocation, a_1 is assigned to i_1 with a compensation of 4, and a_2 to i_2 with a compensation of 2. In the institution-optimal stable allocation, the same assignment prevails, but both agents receive zero compensation. After the score adjustment, the assignment remains unchanged; in the agent-optimal stable allocation, a_1 and a_2 each receive a compensation of 2, while the institution-optimal stable allocation is unaffected. No minority agent experiences a reduction in welfare.

The key distinction between the two market structures lies in the role of alternatives in shaping the allocation. In NTU markets, an institution's outside options have no impact on the welfare of the agents assigned to it. By contrast, in TU markets, the compensation received by an agent depends on the institution's alternatives. In the example, agent a_3 is willing to be assigned to either institution at zero compensation, yielding profits of 2 and 5 for i_1 and i_2, respectively. After the score adjustment, these potential profits increase to 4 and 7, reflecting the intensification of competition among workers. Consequently, while uniform transfers prevent a reduction in the welfare of minority agents in TU markets, certain score adjustments—even if uniform—may reduce minority welfare in NTU markets. Example 2 further illustrates this point by showing that introducing a subsidy for

w_5, in Table 9, lowers the salaries of minority workers in the TU market, whereas the ranking of workers remains unchanged in an NTU market. This demonstrates that a transfer can reduce the welfare of minority workers in a TU market while having no effect in an NTU market.

A.2 Limits to the Substitutability Between Uniform Taxes and Subsidies

In Section 5.3, we show that uniform transfers permit a substitution between taxes and subsidies when revenue functions are additively separable. In this appendix, we demonstrate that this substitution is not possible for more general revenue functions within the class of demand functions satisfying the gross substitutes condition. A necessary condition for such a substitution is that firms' demand sets remain invariant under either transfer. Under additively separable revenue functions, each worker can be evaluated independently, and the transfer amounts to an identical adjustment—positive or negative—to the revenue generated by every worker within a group. For any subset of workers, the marginal contribution of each worker consists of three components—the revenue they generate individually, the valuation term capturing their preference for the firm, and the transfer. By contrast, when the marginal revenue generated by workers is lower than their individual revenues, taxes and subsidies may have asymmetric effects. The following example illustrates this asymmetry.

Example 4. Consider a market consisting of a single firm, $F = \{f\}$, and two workers, $W = \{w_1, w_2\}$. Suppose that $\sigma_{w,f} = 0$ for each $w \in W$. Let the revenue function be such that $R_f(\{w_1\}) = R_f(\{w_2\}) = x$ and $R_f(\{w_1, w_2\}) = 2x - \varepsilon$, with $x > \varepsilon > 0$. Suppose that $M = \{w_1\}$ and $m = \{w_2\}$, and consider the salary vector $\mathbf{s} = (0, 0)$. It is clear that $D_f(\mathbf{s}, R_f, \mathbf{T}_0) = \{\{w_1, w_2\}\}$.
Now consider two uniform transfers $\mathbf{T}$ and $\mathbf{T}'$ such that $\mathbf{T}$ subsidizes minority workers by $t = x$, and $\mathbf{T}'$ taxes majority workers by $t' = -x$, so that $t = |t'|$. Moreover, for all $w \in W$, $R_f(\{w\}) + \sigma_{w,f} + t'_{w,f} \geq 0$. The corresponding demand sets are:

$$D_f(\mathbf{s}, R_f, \mathbf{T}) = \{\{w_1, w_2\}\} \text{ and } D_f(\mathbf{s}, R_f, \mathbf{T}') = \{\{w_2\}\}.$$

The two demand sets differ because the marginal revenue of w_1 under transfer $\mathbf{T}'$ is given by:

$$R_f(\{w_1, w_2\}) + t'_{w_1,f} - (R_f(\{w_2\})) = -\varepsilon.$$

Thus, although both transfers are uniform and identical in magnitude, they induce distinct demand responses from the firm. This illustrates that when revenue functions exhibit submodularity—i.e., the marginal contribution of a worker decreases in the presence of others—taxes and subsidies may not be substitutable, even under uniform transfers.

B Algorithms

This section presents two algorithms implementing the mechanisms discussed in this article. We first describe the ascending salary adjustment process, which produces a firm-optimal stable allocation for any market and is naturally defined for markets with capacities, as only firms' demand sets are

adjusted at each step. We then describe the descending salary adjustment process for markets with capacities, as considered in Section 5. For simplicity, proposed salaries increase by one unit at each step in the ascending salary adjustment process, while utilities decrease by one unit at each step in the descending salary adjustment process, following Kelso and Crawford (1982).

B.1 Ascending Salary Adjustment Process

This section introduces the *ascending salary adjustment process* (Kelso and Crawford 1982) for a market $G \in \mathbb{G}$ and for any transfer $\mathbf{T} \in \mathbb{T}$.

Step 0. For each firm $f \in F$, consider an initial vector of salaries $\mathbf{s}_f(0) \equiv (-\sigma_{w,f})_{w \in W}$, and let $W'_f(0) \subset W$ be a subset of workers such that $W'_f(0) \in D_f(\mathbf{s}_f(0); R_f, \mathbf{T})$, i.e., a subset that maximizes the profit of firm f under the current vector of salaries. If there is a tie between workers, a tie-breaking rule is imposed. For each worker $w \in W'_f(0)$, firm f offers the salary $\mathbf{s}_f(0)(w)$ to each worker $w \in W'_f(k)$ (i.e., $-\sigma_{w,f}$).
Step $k \geq 1$. If a worker receives more than one offer, she rejects all offers except her most preferred one, which she tentatively accepts. In case of a tie, a tie-breaking rule is imposed. For each firm f, if a worker w rejects the offer, the salary for w is increased by 1, i.e.,

$$\mathbf{s}_f(k)(w) = \mathbf{s}_f(k-1)(w) + 1.$$

Next, consider the updated vector of salaries $\mathbf{s}_f(k)$, and let $W'_f(k) \subset W$ be the subset of workers such that $W'_f(k) \in D_f(\mathbf{s}_f(k); R_f, \mathbf{T})$, i.e., the subset that maximizes the profit of firm f at step k, given the current vector of salaries. If there is a tie, a tie-breaking rule is imposed. Firm f then offers the updated salary $\mathbf{s}_f(k)(w)$ to each worker $w \in W'_f(k)$.
The algorithm terminates if no worker rejects an offer.

The process continues until at least one offer is rejected. Since the number of firms is finite and each firm's profit cannot be negative, the algorithm terminates in a finite number of steps. Workers who have not rejected the salary offers accept them and are employed by the firms making the offers. The final outcome is a firm-optimal stable allocation.

B.2 Descending Salary Adjustment Process

This subsection introduces the *descending salary adjustment process* for markets with capacities and for any transfer $\mathbf{T} \in \mathbb{T}$.

Step 0. For each worker $w \in W$, consider an initial vector of expected utility $\mathbf{u}_w(0) \equiv (R_f(w) + \sigma_{w,f} + t_{w,f})_{f \in F}$. Each worker selects a firm f such that

$$\mathbf{u}_w(0)(f) = \max_{f \in F}(\mathbf{u}_w(0)(f)) \geq 0,$$

i.e., the firm that offers the highest expected utility. If there is a tie between firms, a tie-breaking

rule is imposed. Each worker w then proposes a salary that yields utility $\mathbf{u}_w(0)(f)$ to that firm.
Step $k \geq 1$. If a firm f receives more proposals than its capacity q_f, it rejects all proposals. For each firm $f \in F$ that rejects all its proposals, all workers $w \in W$ reduce their expected utility at that firm by 1, i.e.,

$$\mathbf{u}_w(k)(f) = \mathbf{u}_w(k-1)(f) - 1.$$

Each worker then selects a firm f such that

$$\mathbf{u}_w(k)(f) = \max_{f \in F}(\mathbf{u}_w(k)(f)) \geq 0.$$

If there is a tie between firms, a tie-breaking rule is imposed. Each worker w proposes a salary of $\mathbf{u}_w(k)(f)$ to that firm.
If no such firm exists, worker w proposes to the null firm $\varnothing$, and remains unemployed.
The algorithm continues while at least one proposal is rejected. Since the number of workers and firms is finite, and utility cannot become negative, the algorithm terminates in a finite number of steps. Firms that have not rejected the salary offers accept them and employ the workers who proposed them. The final outcome is a worker-optimal stable allocation.

C Auxiliary Results

Lemma 1 provides a decomposition of transfers, allowing us to analyze their effects on minority workers' welfare. The intuition is that for a given market $G \in \mathbb{G}$ and a transfer $\mathbf{T}$, the transfer can be decomposed into several matrices whose sum equals $\mathbf{T}$.

Lemma 1. Consider a market $G \in \mathbb{G}$, a transfer $\mathbf{T} \in \mathbb{T}$, and $\mathbb{T}' \subset \mathbb{T}$ such that $\mathbf{T} = \sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i$. Then, there exist worker-optimal and firm-optimal stable allocations in the markets $(\mathbf{T})$ and $(\sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i)$ such that:

$$(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) = (\mu_W^{\sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i}, \mathbf{s}_W^{\sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i}) \text{ and } (\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) = (\mu_F^{\sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i}, \mathbf{s}_F^{\sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i}).$$

For convenience, we denote by $(\mathbf{T}_0) \xrightarrow{\mathbf{T}} (\mathbf{T})$ the *implementation of the transfer* $\mathbf{T}$ in a market $(\mathbf{T}_0)$, with $(\mathbf{T})$ as the resulting market. By decomposing $\mathbf{T}$, there exists $\mathbb{T}' \subset \mathbb{T}$ such that $\mathbf{T} = \sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i$, and

$$(\mathbf{T}_0) \xrightarrow{\mathbf{T}_1} (\mathbf{T}_1) \xrightarrow{\mathbf{T}_2} (\mathbf{T}_1 + \mathbf{T}_2) \xrightarrow{\mathbf{T}_3} \cdots \xrightarrow{\mathbf{T}_\ell} \left(\sum_{\mathbf{T}_i \in \mathbb{T}'} \mathbf{T}_i \right),$$

where ℓ denotes the index of the last transfer implemented in the sequence. This decomposition primarily facilitates the analysis of the impact of transfers on workers. For instance, if $\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2$, and both $\mathbf{T}_1$ and $\mathbf{T}_2$ do not reduce the welfare of minority workers, then $\mathbf{T}$ does not reduce the welfare of minority workers either. Similarly, it follows that Lemma 2 captures the maximum impact of each transfer.

Lemma 2. Consider a market $G \in \mathbb{G}$, and a transfer $\mathbf{T} \in \mathbb{T}$ that subsidizes minority workers. Then, for each worker $w \in W$,

(i) $u_w(\mu_F, \mathbf{s}_F) - u_w(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) \leq \max(\mathbf{T})$, and
(ii) $u_w(\mu_W, \mathbf{s}_W) - u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \leq \max(\mathbf{T})$.

Proof. (i) Let $(\mu_F, \mathbf{s}_F)$ be the firm-optimal stable allocation, and consider a transfer $\mathbf{T}$ such that $t_{w,f} \geq 0$ for all (w, f), and $t_{w,f} > 0$ only if $w \in m$. Suppose, for contradiction, that there exists a worker $w \in W$ such that

$$u_w(\mu_F, \mathbf{s}_F) - u_w(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) > \max(\mathbf{T}).$$

Since $\max(\mathbf{T}) \geq 0$ and $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$ satisfies individual rationality, it follows that $u_w(\mu_F, \mathbf{s}_F) > \max(\mathbf{T})$. Because $(\mu_F, \mathbf{s}_F)$ is the firm-optimal stable allocation, and $u_w(\mu_F, \mathbf{s}_F) > 0$, there exists a firm $f \neq \mu_F(w)$ and a salary $s_{w,f}$ such that $u_w(\mu_F, \mathbf{s}_F) = u_w(f, s_{w,f})$. Thus, w belongs to a demand set for both $\mu_F(w)$ and f at salaries $\mathbf{s}_{Fw,\mu_F(w)}$ and $s_{w,f}$, respectively.
When $\mathbf{T}$ is introduced, firm profits increase by at most $\max(\mathbf{T})$ per worker. Furthermore, by the gross substitutes condition, reducing a worker's salary by up to $\max(\mathbf{T})$ does not remove that worker from any firm's demand set.[34]
Thus, at salaries $\mathbf{s}_{Fw,\mu_F(w)} - \max(\mathbf{T})$ and $s_{w,f} - \max(\mathbf{T})$, worker w remains in a demand set for both $\mu_F(w)$ and f. Hence, w's utility can decrease by at most $\max(\mathbf{T})$ after the introduction of $\mathbf{T}$. Therefore,

$$u_w(\mu_F, \mathbf{s}_F) - u_w(\mu_F^{\mathbf{T}}, s_F^{\mathbf{T}}) \leq \max(\mathbf{T}).$$

This concludes the proof of part (i).
(ii) Let $(\mu_W, \mathbf{s}_W)$ be the worker-optimal stable allocation, and consider a transfer $\mathbf{T}$ such that $t_{w,f} \geq 0$ for all (w, f), and $t_{w,f} > 0$ only if $w \in m$. Suppose, for contradiction, that there exists a worker $w \in W$ such that

$$u_w(\mu_W, \mathbf{s}_W) - u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) > \max(\mathbf{T}).$$

As in part (i), it follows that $u_w(\mu_W, \mathbf{s}_W) > \max(\mathbf{T})$. By definition of the worker-optimal stable allocation, no worker $w' \in W$ and no firm $\mu_W(w') \neq \mu_W(w)$ exist together with a salary vector $\mathbf{s}$ such that

$$u_{w'}(\mu_W(w), s_{w',\mu_W(w)}) > u_{w'}(\mu_W, \mathbf{s}_W),$$

and

$$V_{\mu_W(w)}(W', \mathbf{s}, R_{\mu_W(w)}, \mathbf{T}_0) \geq V_{\mu_W(w)}(\mu_W, \mathbf{s}_W, R_{\mu_W(w)}, \mathbf{T}_0),$$

for some W' containing w'.
Following similar logic to part (i), after introducing $\mathbf{T}$, each firm's profit increases by at most $\max(\mathbf{T})$ per worker. The firms' demand satisfies the gross substitutes condition. Thus, if the salary of worker w is reduced to $s_{Ww,\mu_W(w)} - \max(\mathbf{T})$, w will remain in the demand set of $\mu_W(w)$, as no worker is subsidized by more than $\max(\mathbf{T})$. Therefore,

$$u_w(\mu_W, \mathbf{s}_W) - u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \leq \max(\mathbf{T}),$$

[34] Reducing salaries weakly expands a firm's demand set (see Kelso and Crawford 1982; Hatfield and Milgrom 2005; Echenique 2012).

which contradicts the assumption.
This completes the proof. ■

The following lemma, established by Sotomayor (1999), follows from the genericity assumption and shows that the assignment is unique across stable allocations.[35]

Lemma 3 (Sotomayor 1999)**.** Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. Generically, for any two stable allocations (μ, s) and (μ', s'), we have $\mu = \mu'$.

Lemma 4 provides the expression for worker utility at worker-optimal stable allocation.

Lemma 4. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$. In any worker-optimal stable allocation $(\mu_W, \mathbf{s}_W)$, for each firm $f \in F$ and any pair of workers $w, w' \in \mu_W(f)$, it holds that:

$$R_f(w) - s_{Ww,f} = R_f(w') - s_{Ww',f}.$$

Proof. If $w = w'$, the statement is trivial. Now suppose $w \neq w'$ and, for contradiction, assume that:

$$R_f(w) - s_{Ww,f} \neq R_f(w') - s_{Ww',f}.$$

Without loss of generality, suppose that:

$$R_f(w) - s_{Ww,f} > R_f(w') - s_{Ww',f}.$$

Since $(\mu_W, \mathbf{s}_W)$ is the worker-optimal stable allocation, workers receive their highest possible utility over all stable allocations, there does not exist any worker $w'' \notin \mu_W(f)$ and salary vector $\mathbf{s}'$ such that

$$V_f((\mu_W(f) \cup \{w''\}) \setminus \{w'\}, \mathbf{s}', R_f, \mathbf{T}, \mathbf{q}) \geq V_f(\mu_W, \mathbf{s}_W, R_f, \mathbf{T}, \mathbf{q}),$$

and,

$$u_{w''}(f, s'_{w'',f}) > u_{w''}(\mu_W, \mathbf{s}_W).$$

Hence, there exists a salary vector $\mathbf{s}'$ such that $s'_{w',f} = s_{Ww',f}$ for every $w' \in W \setminus \{w\}$, and $s'_{w,f} > s_{Ww,f}$ under which $(\mu_W, \mathbf{s})$ is not blocked. This contradicts the worker-optimality of $(\mu_W, \mathbf{s}_W)$. Therefore, we must have:

$$R_f(w) - s_{Ww,f} = R_f(w') - s_{Ww',f} \quad \text{for all } w, w' \in \mu_W(f).$$

■

D Proofs

For clarity, proofs are not presented in the order of appearance of the results in the main text. Instead, they are organized according to logical dependence, beginning with results that form the

[35] This result follows from the First Welfare Theorem: all equilibrium allocations, which correspond to stable allocations in this setting, are efficient in the sense that they maximize total surplus, defined as the sum of firms' profits and workers' utilities.

foundation for subsequent arguments.

D.1 Proof of Proposition 1

Proof. Consider a market $G \in \mathbb{G}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} < 0$ only if $w \in M$, and $t_{w,f} = 0$ otherwise. The proof proceeds by analyzing the ascending salary adjustment process under $\mathbf{T}$.

At initialization, define for each firm f the salary vector $\mathbf{s}_f(0) \equiv (-(\sigma_{w',f} + t_{w',f}))_{w' \in W}$. By construction, for all $w \in W \cap m$ and all $f \in F$, $\mathbf{s}_f^{\mathbf{T}}(0)(w) = \mathbf{s}_f(0)(w)$, while for some $w' \in W \cap M$, $\mathbf{s}_f^{\mathbf{T}}(0)(w') > \mathbf{s}_f(0)(w')$. Consequently, $\mathbf{s}_f^{\mathbf{T}}(0) \geq \mathbf{s}_f(0)$.

Since firms' demand correspondences satisfy the gross substitutes condition, it follows that if $W' \in D_f(\mathbf{s}_f(0), R_f, \mathbf{T}_0)$, with $w \in W'$, then there exists $W'' \in D_f(\mathbf{s}_f^{\mathbf{T}}(0), R_f, \mathbf{T})$ such that $w \in W''$.

Moreover, salaries are normalized by adjusting for the transfers, so that the effective salary offered to each worker w at step 0 is $\mathbf{s}_f^{\mathbf{T}}(0)(w) + t_{w,f}$. Under these adjustments, minority workers receive weakly more offers at each step of the process.

By monotonicity of workers' preferences in salaries, each minority worker attains weakly higher utility under the allocation $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$ than under $(\mu_F, \mathbf{s}_F)$. ■

D.2 Proof of Theorem 2

Proof. Consider a market $G \in \mathbb{G}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$, and for each $w \in m$, $t_{w,\mu_W(w)} = \max_{f \in F}(t_{w,f})$.

By Lemma 1, $\mathbf{T}$ can be decomposed as

$$\mathbf{T} = \sum_{w \in W} \mathbf{T}^w,$$

where each $\mathbf{T}^w \in \{\mathbf{T}^w\}_{w \in W} \subset \mathbb{T}$ affects only worker w, with $t_{w,f}$ as in $\mathbf{T}$, i.e., for each $f \in F$, $t^w_{w,f} = t_{w,f}$, and all other elements are zero.

Fix $w \in W$, and consider $\mathbf{T}^w$. We first show that under $\mathbf{T}^w$, worker w's utility at the worker-optimal stable allocation weakly increases by at least $t_{w,\mu_W(w)}$.

Since $(\mu_W, \mathbf{s}_W)$ is the worker-optimal stable allocation under $\mathbf{T}_0$, no allocation $(\mu', \mathbf{s}')$ satisfies

$$u_w(\mu', \mathbf{s}') > u_w(\mu_W, \mathbf{s}_W) \quad \text{and} \quad V_{\mu'(w)}(\mu'; \mathbf{s}', R_{\mu'(w)}, \mathbf{T}_0) \geq V_{\mu_W(w)}(\mu_W; \mathbf{s}_W, R_{\mu_W(w)}, \mathbf{T}_0).$$

By stability, the profit of firm $\mu_W(w)$ under $(\mu_W, \mathbf{s}_W)$ cannot be increased without decreasing the utility of some workers.

Introducing $\mathbf{T}^w$ shifts the profit of firm $\mu_W(w)$ by $t_{w,\mu_W(w)}$, yielding

$$V_{\mu_W(w)}(\mu_W; \mathbf{s}_W, R_{\mu_W(w)}, \mathbf{T}^w) = V_{\mu_W(w)}(\mu_W; \mathbf{s}_W, R_{\mu_W(w)}, \mathbf{T}_0) + t_{w,\mu_W(w)}.$$

For each worker $w' \in m$, define the salary vector $\mathbf{s}^{\mathbf{T}^w}$ by:

$$s^{\mathbf{T}^w}_{w',\mu_W(w')} = \begin{cases} s_{Ww',\mu_W(w')} + t_{w',\mu_W(w)} & \text{if } w' = w, \\ s_{Ww',\mu_W(w')} & \text{otherwise.} \end{cases}$$

By construction,

$$V_{\mu_W(w)}(\mu_W; \mathbf{s}^{\mathbf{T}^w}, R_{\mu_W(w)}, \mathbf{T}^w) = V_{\mu_W(w)}(\mu_W; \mathbf{s}_W, R_{\mu_W(w)}, \mathbf{T}_0),$$

which guarantees that stability is preserved at $\mu_W(w)$ under $(\mu_W, \mathbf{s}^{\mathbf{T}^w})$ with the transfer $\mathbf{T}^w$. Moreover, since the largest transfer $t_{w,\mu_W(w)}$ is received at $\mu_W(w)$, worker w has no incentive to deviate to another firm. Otherwise, this would contradict that $(\mu_W, \mathbf{s}_W)$ is the worker-optimal stable allocation. Thus, the assignment remains unchanged: $\mu_W^{\mathbf{T}^w}(w) = \mu_W(w)$. Furthermore, in the allocation $(\mu_W, \mathbf{s}^{\mathbf{T}^w})$, worker w's utility increases exactly by $t_{w,\mu_W(w)}$.
Repeating this argument for each $w \in m$, and recalling that $\mathbf{T} = \sum_{w \in W} \mathbf{T}^w$, it follows that under the transfer $\mathbf{T}$, the salary vector $\mathbf{s}^{\mathbf{T}}$ defined by

$$s^{\mathbf{T}}_{w',\mu_W(w')} = s_{Ww',\mu_W(w')} + t_{w',\mu_W(w')}, \text{ for all } w \in W,$$

yields a stable allocation $(\mu_W, \mathbf{s}^{\mathbf{T}})$. Therefore, for each $w \in m$,

$$u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \geq u_w(\mu_W, \mathbf{s}^{\mathbf{T}}),$$

with an improvement of at least $t_{w,\mu_W(w)}$ and $\mu_W^{\mathbf{T}} = \mu_W$. ■

D.3 Proof of Theorem 1

We begin by showing that a uniform transfer never reduces the welfare of minority workers under both φ_F and φ_W.

Proof. Consider a market $G \in \mathbb{G}$ and a uniform transfer $\mathbf{T} \in \mathbb{T}$ such that for each $w \in m$ and each $f \in F$, $t_{w,f} = t \geq 0$, and for each $w' \in M$ and each $f \in F$, $t_{w',f} = t' \leq 0$. By Lemma 1, we decompose $\mathbf{T}$ into two transfers as follows:

- $\mathbf{T}^m$ is defined such that for each $w \in m$ and each $f \in F$, $t^m_{w,f} = t_{w,f}$, and all other elements are zero;
- $\mathbf{T}^M$ is defined such that for each $w \in M$ and each $f \in F$, $t^M_{w,f} = t_{w,f}$, and all other elements are zero.

We first examine $\mathbf{T}^M$. From Proposition 1, we know that taxing majority workers does not reduce the welfare of minority workers under the firm-optimal stable allocation. Thus, for each $w \in m$,

$$u_w(\mu_F^{\mathbf{T}^M}, \mathbf{s}_F^{\mathbf{T}^M}) \geq u_w(\mu_F, \mathbf{s}_F).$$

Consider now the worker-optimal stable allocation $(\mu_W, \mathbf{s}_W)$. By definition, $(\mu_W, \mathbf{s}_W)$ yields the highest utility to each worker among all stable allocations. In particular, for any $w \in W$, no firm $f \in F \setminus \{\mu_W(w)\}$ offers a strictly higher salary without decreasing firm profits, that is, there does

not exist $f \neq \mu_W(w)$ and $\mathbf{s}$ such that

$$u_w(f, s_{w,f}) > u_w(\mu_W, \mathbf{s}_W),$$

and simultaneously

$$V_f(W', \mathbf{s}, R_f, \mathbf{T}_0) \geq V_f(\mu_W, \mathbf{s}_W, R_f, \mathbf{T}_0),$$

for some W' containing w.

Introducing $\mathbf{T}^M$, we know that for all firms $f \in F$ and all majority workers $w' \in M$, the transfer is $t^M_{w',f} = t' \leq 0$. Since the tax applies to all firms, it follows that for each majority worker, the highest utility w' can achieve at any stable allocation, when employed, is

$$\sigma_{w',\mu_W(w')} + s_{Ww',\mu_W(w')} + t'.$$

As in the proof of Proposition 1, consider a salary vector $\mathbf{s}'$ defined by

$$s^{\mathbf{T}^M}_{w,\mu_W(w)} = s_{Ww',\mu_W(w')} - t^M_{w,\mu_W(w)},$$

for each $w \in W$. Since $t' \leq 0$, it follows that $\mathbf{s}^{\mathbf{T}^M} \geq \mathbf{s}_W$. Because firms' demands satisfy the gross substitutes condition, we know that for each minority worker $w \in m$, w remains in a demand set in $D_{\mu_W(w)}(\mathbf{s}^{\mathbf{T}^M}, R_{\mu_W(w)}, \mathbf{T}^M)$. Therefore, at the worker-optimal stable allocation, for each minority worker $w \in m$,

$$u_w(\mu_W^{\mathbf{T}^M}, \mathbf{s}_W^{\mathbf{T}^M}) \geq u_w(\mu_W, \mathbf{s}_W).$$

We now consider $\mathbf{T}^m$ and the worker-optimal stable allocation. We decompose the transfer $\mathbf{T}^m$ into two transfers as follows:

- $\mathbf{T}^{m_E}$ is defined such that for each $w \in m$ with $\mu_W(w) \neq \varnothing$, for each $f \in F$, $t^{m_E}_{w,f} = t_{w,f} = t$, and all other elements are zero;
- $\mathbf{T}^{m_U}$ is defined such that for each $w \in m$ with $\mu_W(w) = \varnothing$, for each $f \in F$, $t^{m_U}_{w,f} = t_{w,f} = t$, and all other elements are zero.

From Theorem 2, it follows that for each $w \in W$,

$$u_w(\mu_W^{\mathbf{T}^{m_E}}, \mathbf{s}_W^{\mathbf{T}^{m_E}}) \geq u_w(\mu_W, \mathbf{s}_W) + t^{m_E}_{w,\mu_W(w)}.$$

By Lemma 2 (ii), for each $w \in W$,

$$u_w(\mu_W^{\mathbf{T}^{m_E}}, \mathbf{s}_W^{\mathbf{T}^{m_E}}) - u_w(\mu_W^{(\mathbf{T}^{m_E}+\mathbf{T}^{m_U})}, \mathbf{s}_W^{(\mathbf{T}^{m_E}+\mathbf{T}^{m_U})}) \leq \max(\mathbf{T}^{m_U}) = t.$$

It follows that for each $w \in m$ such that $\mu_W(w) \neq \varnothing$,

$$u_w(\mu_W^{\mathbf{T}^m}, \mathbf{s}_W^{\mathbf{T}^m}) \geq u_w(\mu_W, \mathbf{s}_W).$$

For each $w \in m$ such that $\mu_W(w) = \varnothing$, we have $u_w(\mu_W, \mathbf{s}_W) = 0$, and since $(\mu_W^{\mathbf{T}^m}, \mathbf{s}_W^{\mathbf{T}^m})$ is individ-

ually rational, it follows that $u_w(\mu_W^{\mathbf{T}^m}, \mathbf{s}_W^{\mathbf{T}^m}) \geq 0$. Therefore, for every minority worker $w \in m$,

$$u_w(\mu_W^{\mathbf{T}^m}, \mathbf{s}_W^{\mathbf{T}^m}) \geq u_w(\mu_W, \mathbf{s}_W).$$

It remains to show that $\mathbf{T}^m$ does not reduce the welfare of minority workers at the firm-optimal stable allocation. Suppose, for contradiction, that there exists $w \in m$ such that

$$u_w(\mu_F, \mathbf{s}_F) > u_w(\mu_F^{\mathbf{T}^m}, \mathbf{s}_F^{\mathbf{T}^m}).$$

If $u_w(\mu_F, \mathbf{s}_F) = 0$, the contradiction is immediate, as individual rationality ensures

$$u_w(\mu_F^{\mathbf{T}^m}, \mathbf{s}_F^{\mathbf{T}^m}) \geq 0,$$

and thus

$$u_w(\mu_F^{\mathbf{T}^m}, \mathbf{s}_F^{\mathbf{T}^m}) \geq u_w(\mu_F, \mathbf{s}_F).$$

Suppose instead that $u_w(\mu_F, \mathbf{s}_F) > 0$. Then, by the optimality of $(\mu_F, \mathbf{s}_F)$, there exists a firm $f \in F$, $f \neq \mu_F(w)$, and a salary vector $\mathbf{s}$ such that

$$u_w(f, s_{w,f}) = u_w(\mu_F, \mathbf{s}_F),$$

and

$$V_f(W'; \mathbf{s}, R_f, \mathbf{T}_0) = V_f(\mu_F; \mathbf{s}_F, R_f, \mathbf{T}_0),$$

for some W' containing w.

From Lemma 1, we can further decompose $\mathbf{T}^m$ into two transfers, $\{\mathbf{T}_w^m, \mathbf{T}_{-w}^m\} \subset \mathbb{T}$, where $\mathbf{T}_w^m$ consists only of the transfers received by w, i.e., for each $f \in F$, $t^m_{w_{w,f}} = t^m_{w,f}$ and zero for other elements, and $\mathbf{T}_{-w}^m$ consists of the transfers received by all other workers, i.e., for each $w' \in W \setminus \{w\}$, for each $f \in F$, $t^m_{-w_{w',f}} = t^m_{w',f}$, and $t_{-w_{w,f}} = 0$.

Consider $(\mathbf{T}_0) \overset{\mathbf{T}_w^m}{\rightarrow} (\mathbf{T}_w^m)$. It is direct that firm f can offer a salary of $s^t_{w,f} = s_{w,f} + t$ while

$$V_f(W'; (s^t_{w,f}, \mathbf{s}_{-w}), R_f, \mathbf{T}_0) = V_f(W'; \mathbf{s}, R_f, \mathbf{T}_0).$$

Similarly, $\mu_F(w)$ can offer a salary of $s_{w,\mu_F(w)} + t$ while also preserving its profit. Therefore, the utility of w increases by t at the firm-optimal stable allocation $(\mu_F^{\mathbf{T}_w^m}, \mathbf{s}_F^{\mathbf{T}_w^m})$, that is

$$u_w(\mu_F^{\mathbf{T}_w^m}, \mathbf{s}_F^{\mathbf{T}_w^m}) = u_w(\mu_F, \mathbf{s}_F) + t.$$

We then introduce $\mathbf{T}_{-w}^m$, such that $(\mathbf{T}_w^m) \overset{\mathbf{T}_{-w}^m}{\rightarrow} (\mathbf{T}^m)$ since $\mathbf{T}_w^m + \mathbf{T}_{-w}^m = \mathbf{T}^m$. From Lemma 2 (i), we know that the utility of w decreases by at most $\max(\mathbf{T}_{-w}^m) = t$. Hence,

$$u_w(\mu_F^{\mathbf{T}^m}, \mathbf{s}_F^{\mathbf{T}^m}) \geq u_w(\mu_F, \mathbf{s}_F) + t - t = u_w(\mu_F, \mathbf{s}_F),$$

contradicting the assumption. The proof is complete. ■

It remains to show that if a transfer $\mathbf{T}$ is not uniform, then there exists a market in which $\mathbf{T}$ reduces the welfare of minority workers under either φ_F or φ_W. It suffices to demonstrate that $\mathbf{T}$ reduces the welfare of minority workers under φ_W in a single market.

Proof. Consider a market with one firm, $F = \{f\}$, and two workers, $W = \{w_1, w_2\}$, with $\sigma_{w,f} = 0$ for all $w \in W$. Let the group partition be $\mathcal{P}$ such that all workers are minorities: $m = W$, $M = \emptyset$. The firm's revenue function is defined such that for each $W' \subseteq W$,

$$R_f(W') = \begin{cases} R_f(\{w\}), & \text{if } W' = \{w\} \text{ for some } w \in W, \\ 0, & \text{otherwise.} \end{cases}$$

Firm	w_1	w_2
f	x_1	x_2

Table 13: Firm's revenue function.

Suppose $\mathbf{T}$ is not uniform, that is, $t_{w_1,f} \neq t_{w_2,f}$. Without loss of generality, assume $x_2 > x_1$, and suppose $t_{w_1,f} > t_{w_2,f}$, with $x_2 + t_{w_2,f} \geq x_1 + t_{w_1,f}$.
Before the implementation of $\mathbf{T}$, the worker-optimal stable allocation is:

f
w_2
$s_{w_2} = x_2 - x_1$

Table 14: Worker-optimal stable allocation without transfer.

Now, suppose transfer $\mathbf{T}$ is implemented. A worker-optimal stable allocation with transfer $\mathbf{T}$ is given by:

f
w_2
$s_{w_2} = x_2 + t_{w_2,f} - (x_1 + t_{w_1,f})$

Table 15: Worker-optimal stable allocation with transfer $\mathbf{T}$.

Since $t_{w_1,f} > t_{w_2,f}$, it follows directly that

$$x_2 - x_1 > x_2 + t_{w_2,f} - (x_1 + t_{w_1,f}),$$

so that

$$u_{w_2}(\mu_W, \mathbf{s}_W) > u_{w_2}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}).$$

That is, the introduction of $\mathbf{T}$ reduces the welfare of minority worker w_2 under φ_W, completing the proof. ■

D.4 Proof of Theorem 5

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T}, \mathbf{T}' \in \mathbb{T}$ be uniform transfers such that:

- $\mathbf{T}$ subsidizes minority workers by t without taxing majority workers, i.e., $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$, and $t_{w,f} > 0$ only if $w \in m$, and
- $\mathbf{T}'$ taxes majority workers by t' without subsidizing minority workers, i.e., $t_{w,f} \leq 0$ for all $w \in W$ and $f \in F$, and $t_{w,f} < 0$ only if $w \in M$,

such that $t = |t'|$. In addition, for each $f \in F$ and $w \in M$,

$$R_f(w) + t'_{w,f} + \sigma_{w,f} - u_w(\mu_W^{\mathbf{T}'}, \mathbf{s}_W^{\mathbf{T}'}) > 0.$$

Note that this condition implies that, for each $f \in F$ and $w \in M$,

$$R_f(w) + t'_{w,f} + \sigma_{w,f} - u_w(\mu_F^{\mathbf{T}'}, \mathbf{s}_F^{\mathbf{T}'}) > 0,$$

by the definition of the firm-optimal stable allocation.
We consider the construction of the ascending salary adjustment process (Appendix B.1) under both transfers.

Claim 1. At each step k of the ascending salary adjustment process, the set of workers $W'_f(k)$ selected by any firm $f \in F$ is identical under both $\mathbf{T}$ and $\mathbf{T}'$. Moreover, salaries offered to these workers also coincide.

Proof. We implicitly assume that the same tie-breaking rules apply throughout the ascending salary adjustment process. Fix a firm $f \in F$. Since the initial salary vector $\mathbf{s}_f(0)$ is identical in both markets, we compare the firm's initial choice sets under transfers $\mathbf{T}$ and $\mathbf{T}'$. Specifically, we show that the firm selects the same set of workers, that is, $W'_f(0) \in D_f(\mathbf{s}_f(0); R_f, \mathbf{T}, \mathbf{q})$ and $W'_f(0) \in D_f(\mathbf{s}_f(0); R_f, \mathbf{T}', \mathbf{q})$.
Suppose instead that these sets differ. Then, there exist workers w and w' such that $w \in W'_f(0)$ for market $(\mathbf{T})$ and $w \notin W'_f(0)$ for market $(\mathbf{T}')$, while $w' \notin W'_f(0)$ for market $(\mathbf{T})$ and $w' \in W'_f(0)$ for market $(\mathbf{T}')$, since for each worker $w \in W$, $R_F(w) + \sigma_{w,f} + t_{w,f} \geq 0$. In this case, firm f's profit from employing these workers must satisfy:

$$R_f(w) + t_{w,f} - \mathbf{s}_f(0)(w) > R_f(w') + t_{w',f} - \mathbf{s}_f(0)(w') \text{ under } (\mathbf{T}),$$

and,

$$R_f(w') + t'_{w',f} - \mathbf{s}_f(0)(w') > R_f(w) + t'_{w,f} - \mathbf{s}_f(0)(w) \text{ under } (\mathbf{T}').$$

Summing both inequalities yields:

$$R_f(w) + t_{w,f} - \mathbf{s}_f(0)(w) + R_f(w') + t'_{w',f} - \mathbf{s}_f(0)(w') > R_f(w') + t_{w',f} - \mathbf{s}_f(0)(w') + R_f(w) + t'_{w,f} - \mathbf{s}_f(0)(w),$$

a contradiction, since $t + t' = 0$.

It follows that $W'_f(0)$ is the same in both markets. Since the set of offers made is identical, workers make the same acceptance and rejection decisions. Therefore, salary vectors evolve identically across both settings. By induction, this argument extends to all subsequent steps. Moreover, the algorithm continues since for all $f \in F$ and each $w \in W$, $R_f(w) + \sigma_{w,f} + t_{w,f} \geq 0$, and, for all $w' \in M$, $R_f(w) + t'_{w,f} + \sigma_{w,f} - u_w(\mu_F^{\mathbf{T}'}, \mathbf{s}_F^{\mathbf{T}'}) > 0$. That is, the algorithm terminates before the profit generated by a majority worker for any firm becomes zero. ■

Hence, we conclude that $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) = (\mu_F^{\mathbf{T}'}, \mathbf{s}_F^{\mathbf{T}'})$.

We now consider the worker-optimal stable allocation. By Lemma 3, we know that $\mu_F^{\mathbf{T}} = \mu_W^{\mathbf{T}}$. Since $\mu_F^{\mathbf{T}} = \mu_F^{\mathbf{T}'}$, it follows that $\mu_W^{\mathbf{T}} = \mu_W^{\mathbf{T}'}$. To complete the argument, we show that $\mathbf{s}_W^{\mathbf{T}} = \mathbf{s}_W^{\mathbf{T}'}$.
We rely on the construction of the descending salary adjustment process described in Appendix B.2. We begin by showing that, for each worker, the firm they initially propose to under transfer $\mathbf{T}$ is the same as under transfer $\mathbf{T}'$.

Claim 2. For each $w \in W$, let

- $f \in \arg\max_{f \in F}(\mathbf{u}_w(0)(f))$ under transfer $\mathbf{T}$, and
- $f' \in \arg\max_{f \in F}(\mathbf{u}_w(0)(f))$ under transfer $\mathbf{T}'$.

Then $f = f'$.

Proof. We implicitly assume that the same tie-breaking rules apply throughout the descending salary adjustment process. Suppose for contradiction that there exists $w \in W$ such that $f \neq f'$. Without loss of generality, assume that $\{f\} = \arg\max_{f \in F}(\mathbf{u}_w(0)(f))$ under $\mathbf{T}$. Then, under $\mathbf{T}'$, there exists $f^* \in F \setminus \{f\}$ such that

$$R_{f^*}(w) + \sigma_{w,f^*} + t'_{w,f^*} > R_f(w) + \sigma_{w,f} + t'_{w,f}. \tag{2}$$

By definition of f under $\mathbf{T}$, we must have

$$R_f(w) + \sigma_{w,f} + t_{w,f} > R_{f^*}(w) + \sigma_{w,f^*} + t_{w,f^*}. \tag{3}$$

Since $t_{w,f} = t_{w,f^*}$ and $t'_{w,f} = t'_{w,f^*}$, inequalities (2) and (3) are directly contradictory. This completes the proof. ■

As a consequence, under both $\mathbf{T}$ and $\mathbf{T}'$, each worker proposes to the same firm at step 0 of the descending salary adjustment process. For notational convenience, let $\mathbf{u}'_w$ denote the expected utility of worker w in market $(\mathbf{T}')$.
It follows that the resulting expected utility difference for each worker $w \in W$, namely

$$\mathbf{u}_w(0) - \mathbf{u}_w(1) = \mathbf{u}'_w(0) - \mathbf{u}'_w(1),$$

is identical across the two markets.

It then follows that workers make proposals to the same firms at every step of the process, by the same argument, until the descending salary adjustment process terminates. Since, for each $f \in F$ and $w \in M$,

$$R_f(w) + t'_{w,f} + \sigma_{w,f} - u_w(\mu_W^{\mathbf{T}'}, \mathbf{s}_W^{\mathbf{T}'}) > 0,$$

workers' proposed utilities decrease sufficiently during the descending salary adjustment process in the market $(\mathbf{T}')$ to ensure that the resulting allocation is individually rational for firms. Let k denote the final step of the descending salary adjustment process in market $(\mathbf{T}')$. By construction, the transfer satisfies $t_{w,f} = t$ for all f and $w \in m$ in market $(\mathbf{T})$ and $t'_{w,f} = 0$ in market $(\mathbf{T}')$, while for $w \in M$, $t_{w,f} = 0$ in market $(\mathbf{T})$ and $t'_{w,f} = -t$ in market $(\mathbf{T}')$. Consequently, for each worker w and firm $f \in F$,

$$\mathbf{u}_w(0)(f) = \mathbf{u}'_w(0)(f) + t.$$

Since

$$\mathbf{u}_w(0)(f) - \mathbf{u}_w(k)(f) = \mathbf{u}'_w(0)(f) - \mathbf{u}'_w(k)(f),$$

it follows that

$$\mathbf{u}_w(k)(f) = \mathbf{u}'_w(k)(f) + t.$$

Since the assignment is the same under both $\mathbf{T}$ and $\mathbf{T}'$, i.e., $\mu_W^{\mathbf{T}} = \mu_W^{\mathbf{T}'}$, it follows that workers propose to the same firms at each step of the process until it terminates. As a result, the process converges to the same allocation under both transfers. ■

D.5 Proof of Proposition 3

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, and $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$. Furthermore, for each $w \in m_E$,

$$t_{w,\mu_F(w)} = \max_{f \in F}(t_{w,f}) \geq \max_{w' \in m_U, f \in F}(t_{w',f}).$$

From Lemma 1, $\mathbf{T}$ can be decomposed into two transfers:

- $\mathbf{T}_1$: For each $w \in m_E$ and $f \in F$, $t_{1_{w,f}} = t_{w,f}$ and 0 for other elements.
- $\mathbf{T}_2$: For each $w' \in m_U$ and $f \in F$, $t_{2_{w',f}} = t_{w',f}$ and 0 for other elements.

By construction, $t_{1_{w,\mu_W(w)}} = \max_{f \in F}(t_{w,f})$. From Theorem 2, it follows that for each $w \in m_E$, with $(\mathbf{T}_0) \overset{\mathbf{T}_1}{\to} (\mathbf{T}_1)$,

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) \geq u_w(\mu_W, \mathbf{s}_W) + t_{1_{w,\mu_W(w)}}.$$

Next, consider $(\mathbf{T}_1) \overset{\mathbf{T}_2}{\to} (\mathbf{T})$ since $\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2$. From Lemma 2, the impact of $\mathbf{T}_2$ is bounded by $\max(\mathbf{T}_2)$. Because $t_{1_{w,\mu_W(w)}} \geq \max(\mathbf{T}_2)$ for each $w \in m_E$, it follows that

$$u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \geq u_w(\mu_W, \mathbf{s}_W).$$

■

D.6 Proof of Theorem 3

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, and $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$. Furthermore:

- For each $w \in m_E \setminus \{w_m\}$ such that $\mu_W(w) \neq f$, $t_{w,\mu_W(w)} = \max_{f' \in F}(t_{w,f'}) \geq t_{w_m \to f}$,
- For each $w \in m_E$ such that $\mu_W(w) = f$, $t_{w,f} \geq \max_{f' \in F \setminus \{f\}}(t_{w,f'}) + t_{w_m \to f}$,
- For each $w' \in W \setminus (m_E \cup \{w_m\})$ and each $f' \in F$, $t_{w',f'} = 0$, and
- For each $f' \in F \setminus \{f\}$, $t_{w_m,f'} = 0$, and $t_{w_m,f} \geq t_{w_m \to f}$.

We examine the firm-optimal stable allocations $(\mu_F, \mathbf{s}_F)$ and $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$.[36] Without loss of generality, let w_M denote the unique majority worker at f who generates the lowest profit, that is,

$$\{w_M\} = \arg \min_{w \in \mu_F(f) \cap M} \left(R_f(w) - s_{Fw_M,f}\right).$$

We now show that $\mathbf{T}$ increases the representation of minority workers in firm f through the hiring of w_m.

Claim 3. $\mathbf{T}$ increases the representation of minority workers in firm f under any stable mechanism.

Proof. We show that (i) $\mu_F^{\mathbf{T}}(w_m) = f$, and (ii) $(\mu(f) \cap m) \subset (\mu^{\mathbf{T}}(f) \cap m)$.
We recall that:[37]

$$t_{w_m \to f} \equiv R_f(w_M) - s_{Fw_M,f} - (R_f(w_m) + \sigma_{w_m,f}) + R_{\mu_F(w_m)}(w_m) + \sigma_{w_m,\mu_F(w_m)}$$
$$- \max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} \left(R_{\mu_F(w_m)}(w') + \sigma_{w',\mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F)\right).$$

(i) Firm f hires w_m under $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$ if

$$R_f(w_m) + t_{w_m \to f} - s_{Fw_m,f}^{\mathbf{T}} \geq R_f(w_M) - s_{Fw_M,f}. \tag{4}$$

From the stability of $(\mu_F, \mathbf{s}_F)$,

$$R_f(w_M) - s_{Fw_M,f} \geq R_f(w_m) + \sigma_{w_m,f} - u_{w_m}(\mu_F, \mathbf{s}_F), \tag{5}$$

which implies

$$u_{w_m}(\mu_F, \mathbf{s}_F) \geq R_f(w_m) + \sigma_{w_m,f} - (R_f(w_M) - s_{Fw_M,f}). \tag{6}$$

Now consider the maximum utility that w_m can obtain at her current firm $\mu_F(w_m)$. The surplus generated by w_m at this firm is $R_{\mu_F(w_m)}(w_m) + \sigma_{w_m,\mu_F(w_m)}$. To substitute w_m, the firm must attract

[36] Our analysis concentrates on the firm-optimal stable allocation for two reasons. First, under the worker-optimal stable allocation, all workers employed by a given firm yield the same profit (see Lemma 4), making it impossible to differentiate among them. Second, under the firm-optimal stable allocation, each firm secures its highest achievable profit across all stable outcomes. Consequently, any decrease in a worker's salary implies the worker has a strictly better outside option, either through employment at another firm or by remaining unemployed.

[37] For notational simplicity, we omit the term $u_{w_m}(\mu_F, \mathbf{s}_F)$, as it does not affect the subsequent calculations.

another worker—assumed here to be either unemployed or w_M—offering that worker at least the utility they currently receive. Hence, the best alternative available to $\mu_F(w_m)$ yields

$$\max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} \left(R_{\mu_F(w_m)}(w') + \sigma_{w', \mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F) \right).$$

Therefore, the maximum utility that w_m can extract at $\mu_F(w_m)$ is

$$R_{\mu_F(w_m)}(w_m) + \sigma_{w_m, \mu_F(w_m)} - \max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} \left(R_{\mu_F(w_m)}(w') + \sigma_{w', \mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F) \right). \quad (7)$$

Combining inequalities (6) and (7) gives

$$R_{\mu_F(w_m)}(w_m) + \sigma_{w_m, \mu_F(w_m)} - \max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} \left(R_{\mu_F(w_m)}(w') + \sigma_{w', \mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F) \right) \geq$$
$$u_{w_m}(\mu_F, \mathbf{s}_F) \geq R_f(w_m) + \sigma_{w_m, f} - (R_f(w_M) - s_{F w_M, f}). \quad (8)$$

It follows that the transfer $t_{w_m \to f}$ renders firm f indifferent between hiring w_m and retaining w_M. Additionally, w_m's utility under $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$ is weakly higher than under the original allocation:

$$u_{w_m}(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) \geq u_{w_m}(\mu_F, \mathbf{s}_F).$$

Now, substituting $t_{w_m \to f}$ into inequality (4) gives

$$R_{\mu_F(w_m)}(w_m) + \sigma_{w_m, \mu_F(w_m)} - \max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} \left(R_{\mu_F(w_m)}(w') + \sigma_{w', \mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F) \right) \geq u_{w_m}(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}). \quad (9)$$

At the firm-optimal stable allocation, by equation (4), firm f has an incentive to recruit w_m, and $\mu_F(w_m)$ is indifferent between recruiting a worker

$$w' \in \arg \max_{w' \in (W_U \cup \{w_M\}) \setminus \{w_m\}} \left(R_{\mu_F(w_m)}(w') + \sigma_{w', \mu_F(w_m)} - u_{w'}(\mu_F, \mathbf{s}_F) \right),$$

and continuing to employ w_m.

Suppose, by contradiction, that $\mu_F^{\mathbf{T}}(w_m) = \mu_F(w_m)$. Then, if firm f hires w_m at salary $s^{\mathbf{T}}_{F w_m, f}$, the profit of f is strictly higher (from equation (4)), and worker w_m obtains weakly higher utility, since $u_{w_m}(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}}) \geq u_{w_m}(\mu_F, \mathbf{s}_F)$, contradicting the stability of the allocation. It follows that $\mu_F^{\mathbf{T}}(w_m) = f$.

We now show that even if there exists a minority worker $w \in m_E \setminus \mu_F(f)$, with $w \neq w_m$, who could be hired by firm f at a subsidy strictly less than $t_{w_m \to f}$, this worker remain assigned to $\mu_F(w)$ under $\mu_F^{\mathbf{T}}$. By the stability of $(\mu_F, \mathbf{s}_F)$, there exists no salary vector $\mathbf{s}$ and set of workers $W' \subseteq W$ such that

$$V_f(W'; \mathbf{s}, R_f, \mathbf{T}_0) > V_f(\mu_F; \mathbf{s}_F, R_f, \mathbf{T}_0), \text{ and } u_w(f, s_{w,f}) \geq u_w(\mu_F, \mathbf{s}_F), \quad (10)$$

for $w \in W'$.

Using the additivity and separability of revenue functions, construct a transfer $\mathbf{T}^w$ such that for all $w' \in W \setminus \{w\}$ and all $f' \in F$, $t^w_{w',f'} = 0$, and $t^w_{w,f'} = t_{w,f'}$. Now, define a salary vector $\mathbf{s}'$ such that, for each $w' \in W$,

$$s'_{w',\mu_F(w')} = s_{Fw',\mu_F(w')} + t^w_{w',\mu_F(w')}.$$

That is, only the salary of w is adjusted, and only by the amount of the transfer.
Since $t_{w,\mu_F(w)} \geq t_{w,f}$ it follows, from (10), that $w \in W'$ for some $W' \in D_{\mu_F(w)}(\mathbf{s}'; R_{\mu_F(w)}, \mathbf{T}^w)$ and $w \notin W''$ for any $W'' \in D_f(\mathbf{s}'; R_f, \mathbf{T}^w)$. That is, under the transfer $\mathbf{T}^w$, worker w remains assigned to $\mu_F(w)$. By the additive separability of revenue functions, the same assignment holds under the transfer $\mathbf{T}$. We conclude that, under $\mu_F^{\mathbf{T}}$, only w_m is hired by firm f.
(ii) From Lemma 1, we decompose $\mathbf{T}$ into three transfers:

- $\mathbf{T}_1$ is defined such that for each $w \in m_E \setminus \{w_m\}$ and $f \in F$, $t_{1_{w,f}} = t_{w,f}$, and 0 for other elements.
- $\mathbf{T}_2$ is defined such that $t_{2_{w_m,f}} = t_{w_m \to f}$, and 0 for other elements.
- $\mathbf{T}_3$ is defined such that $t_{3_{w_m,f}} = t_{w_m,f} - t_{w_m \to f}$, and 0 for other elements.

It is clear that $\mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3 = \mathbf{T}$.
Consider $(\mathbf{T}_0) \overset{\mathbf{T}_1}{\to} (\mathbf{T}_1)$. From Theorem 2, the introduction of $\mathbf{T}_1$ increases the utility of every $w \in \mu_W(f) \cap m$ by at least $t_{1_{w,f}}$, while preserving their assignment; that is $\mu_W^{\mathbf{T}_1}(w) = f$.
Next, $(\mathbf{T}_1) \overset{\mathbf{T}_2}{\to} (\mathbf{T}_1 + \mathbf{T}_2)$, Lemma 2 guarantees that the utility of any worker $w \in \mu_W(f) \cap m$ can decrease by at most $t_{w_m \to f}$. However, since the worker-optimal matching ensures that workers in $\mu_W(f) \cap m$ already attain their highest utility at firm f, and because the constructed transfers satisfy $t_{w,f} \geq \max_{f' \in F \setminus \{f\}}(t_{w,f'}) + t_{w_m \to f}$, these workers remain assigned to firm f in $(\mathbf{T}_1 + \mathbf{T}_2)$. Therefore, for each $w \in \mu_W(f) \cap m$, $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w) = f$. Moreover, from (i), we know that w_m is now assigned to firm f.
When we consider $(\mathbf{T}_1 + \mathbf{T}_2) \overset{\mathbf{T}_3}{\to} (\mathbf{T})$, Theorem 2 ensures that for each $w \in \mu_W^{\mathbf{T}_1+\mathbf{T}_2}(f) \cap m$, $\mu_W^{\mathbf{T}}(w) = f$.
By Lemma 3, since $\mathbf{T}$ increases the representation of minority workers at firm f under φ_F, it also increases the representation of minority workers at firm f under any stable mechanism, which completes the proof. ■

We now show that the transfer $\mathbf{T}$ does not reduce the welfare of minority workers under φ_W.

Claim 4. $\mathbf{T}$ does not reduce the welfare of minority workers under φ_W.

Proof. We rely on the decomposition introduced in the proof of Claim 3 (ii).
First, consider $(\mathbf{T}_0) \overset{\mathbf{T}_1}{\to} (\mathbf{T}_1)$. Since for each $w \in m_E \setminus \{w_m\}$, $t_{1_{w,\mu_W(w)}} = \max_{f \in F}(t_{1_{w,f}}) \geq t_{w_m \to f}$, it follows from Theorem 2 that

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) \geq u_w(\mu_W, \mathbf{s}_W) + t_{1_{w,\mu_W(w)}} \quad \text{for each } w \in m_E \setminus \{w_m\}.$$

Next, consider $(\mathbf{T}_1) \overset{\mathbf{T}_2}{\to} (\mathbf{T}_1 + \mathbf{T}_2)$. From Lemma 2 (ii), for each $w \in m_E \setminus \{w_m\}$,

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) - u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) \leq t_{w_m \to f}.$$

Hence,

$$u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) \geq u_w(\mu_W, \mathbf{s}_W).$$

In addition, Claim 3 implies that $\mu_F^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f$, and by Lemma 3, it follows that $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f$ with $u_{w_m}(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) \geq u_{w_m}(\mu_W, \mathbf{s}_W)$.
Finally, consider $(\mathbf{T}_1 + \mathbf{T}_2) \overset{\mathbf{T}_3}{\to} (\mathbf{T})$, since $\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3$. We know that $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f$, and in $\mathbf{T}_3$, for each $w \in m_E^{\mathbf{T}_1+\mathbf{T}_2}$, $t_{3_{w,\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w)}} = \max_{f\in F}(t_{3_{w,f}})$, by Theorem 2, it follows that

$$u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \geq u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) + t_{3_{w,\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w)}} \geq u_w(\mu_W, \mathbf{s}_W).$$

■

This completes the proof of the result. ■

D.7 Proof of Proposition 4

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, and $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$. Furthermore:

- For each $w \in m_E$ such that $\mu_W(w) = f$, $t_{w,f} \geq \max(0, R_f(w_M) - s_{Fw_M,f} - (R_f(w) - s_{Fw,f}))$,
- For each $w' \in W \setminus (m_E \cap (\mu_F(f) \cup \{w_m\}))$, and each $f' \in F$, $t_{w',f'} = 0$, and
- For each $f' \in F \setminus \{f\}$, $t_{w_m,f'} = 0$, and $t_{w_m,f} \geq t_{w_m \to f}$.

We examine the firm-optimal stable allocations $(\mu_F, \mathbf{s}_F)$ and $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$. From Claim 3, we know that $\mu_F^{\mathbf{T}}(w_m) = f$. It remains to show that for each $w \in m_E \cap \mu_F(f)$, $\mu_F^{\mathbf{T}}(w) = f$.
Without loss of generality, suppose that w_M is the unique majority worker such that $\{w_M\} = \arg\min_{w\in\mu_F(f)\cap M}(R_f(w) - s_{Fw,f})$. By the construction of $\mathbf{T}$ and the optimality of $(\mu_F, \mathbf{s}_F)$ and $(\mu_F^{\mathbf{T}}, \mathbf{s}_F^{\mathbf{T}})$, the assignment $\mu_F^{\mathbf{T}}(w) = f$ for each $w \in m_E \cap \mu_F(f)$ holds if the following inequality is satisfied:

$$R_f(w) + t_{w,f} - s_{Fw,f}^{\mathbf{T}} \geq R_f(w_M) - s_{Fw_M,f}. \tag{11}$$

We now verify this condition by considering two possible cases for each such worker $w \in m_E \cap \mu_F(f)$.

- **Case 1:** Suppose $R_f(w_M) - s_{Fw_M,f} > R_f(w) - s_{Fw,f}$. Then, by construction,

  $$t_{w,f} \geq R_f(w_M) - s_{Fw_M,f} - (R_f(w) - s_{Fw,f}).$$

  Since $(\mu_F, \mathbf{s}_F)$ is stable, there is no firm $f' \neq f$ and salary vector $\mathbf{s}'$ such that $u_w(f', s'_{w,f'}) > u_w(\mu_F, \mathbf{s}_F)$ and $V_{f'}(W', \mathbf{s}', R_{f'}, \mathbf{T}_0, \mathbf{q}) \geq V_{f'}(\mu_F, \mathbf{s}_F, R_{f'}, \mathbf{T}_0, \mathbf{q})$ with $w \in W'$.
  Since $\mathbf{T}$ only affects firm f, firm f continues to offer the highest utility to w under the firm-optimal allocation, implying $s_{Fw,f}^{\mathbf{T}} = s_{Fw,f}$. Therefore, inequality (11) is satisfied.
- **Case 2:** Suppose $R_f(w) - s_{Fw,f} \geq R_f(w_M) - s_{Fw_M,f}$. Then, by definition, $t_{w,f} \geq 0$. As in Case 1, the stability of $(\mu_F, \mathbf{s}_F)$ and the fact that $\mathbf{T}$ only affects firm f implies that $s_{Fw,f}^{\mathbf{T}} = s_{Fw,f}$. Hence,

  $$R_f(w) - s_{Fw,f} \geq R_f(w_M) - s_{Fw_M,f},$$

and inequality (11) again holds.

We conclude that for all $w \in m_E \cap \mu_F(f)$, it must be that $\mu_F^{\mathbf{T}}(w) = f$. Finally, by Lemma 3, since $\mathbf{T}$ increases the representation of minority workers at firm f under φ_F, it also increases their representation at firm f under any stable mechanism, which completes the proof. ■

D.8 Proof of Proposition 5

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, and $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$. Furthermore:

- For each $w \in m_E$, $t_{w,\mu_W(w)} = \max_{f \in F}(t_{w,f}) \geq \max(0, t_{w_m \to m_E} - (s_{Ww,\mu_W(w)} + \sigma_{w,\mu_W(w)}))$,
- For each $w' \in W \setminus (m_E \cup \{w_m\})$ and each $f \in F$, $t_{w',f} = 0$, and
- For each $f \in F \setminus \{f^*\}$, $t_{w_m,f} = 0$, and $t_{w_m,f^*} \geq t_{w_m \to m_E}$.

We first consider the firm-optimal stable allocation $(\mu_F, \mathbf{s}_F)$, and examine the condition under which the unemployed minority worker w_m is hired by firm f^*. Since w_m is unemployed, $u_{w_m}(\mu_F, \mathbf{s}_F) = 0$. At the firm-optimal stable allocation, w_m is hired by f^* if

$$R_{f^*}(w_m) + t_{w,f^*} + \sigma_{w,f^*} \geq R_{f^*}(w_M) - s_{Fw_M,f^*}.$$

where, without loss of generality,

$$\{w_M\} = \arg \min_{w \in \mu_F(f^*) \cap M} (R_{f^*}(w) - s_{Fw,f^*} - R_{f^*}(w_m) - \sigma_{w,f^*}),$$

i.e., w_M is the majority worker generating the lowest profit for firm f^* under $(\mu_F, \mathbf{s}_F)$.

We first establish that subsidizing w_m by $t_{w_m \to m_E}$ induces f^* to hire w_m, as this transfer ensures that w_m generates at least as much profit for f^* as w_M does.

Claim 5. $R_{f^*}(w_m) + t_{w_m \to m_E} + \sigma_{w_m,f^*} \geq R_{f^*}(w_M) - s_{Fw_M,f^*}$.

Proof. By definition,

$$t_{w_m \to m_E} \equiv R_{f^*}(w_M) + \sigma_{w_M,f^*} - (R_{f^*}(w_m) + \sigma_{w_m,f^*}),$$

and hence

$$R_{f^*}(w_m) + t_{w_m \to m_E} + \sigma_{w_m,f^*} = R_{f^*}(w_M) + \sigma_{w_M,f^*}.$$

Since $u_{w_M}(\mu_F, \mathbf{s}_F) = \sigma_{w_M,f^*} + s_{Fw_M,f^*}$ and $(\mu_F, \mathbf{s}_F)$ is individually rational, it follows that $\sigma_{w_M,f^*} \geq -s_{Fw_M,f^*}$, and therefore:

$$R_{f^*}(w_M) + \sigma_{w_M,f^*} \geq R_{f^*}(w_M) - s_{Fw_M,f^*}.$$

■

Next, we show that the utility of w_M under $(\mu_W, \mathbf{s}_W)$ is below the subsidy $t_{w_m \to m_E}$.

Claim 6. $t_{w_m \to m_E} \geq u_{w_M}(\mu_W, \mathbf{s}_W)$.

Proof. Because $(\mu_W, \mathbf{s}_W)$ is stable, and $\mu_W(w_M) = f^* \neq \varnothing$ while $\mu_W(w_m) = \varnothing$, stability implies

$$R_{f^*}(w_M) - s_{Ww_M,f^*} \geq R_{f^*}(w_m) + \sigma_{w_m,f^*}.$$

From the definition of $t_{w_m \to m_E}$,

$$R_{f^*}(w_m) + \sigma_{w_m,f^*} = R_{f^*}(w_M) + \sigma_{w_M,f^*} - t_{w_m \to m_E}.$$

Substituting, we get:

$$R_{f^*}(w_M) - s_{Ww_M,f^*} \geq R_{f^*}(w_M) + \sigma_{w_M,f^*} - t_{w_m \to m_E}.$$

Rewriting the right-hand side as:[38]

$$R_{f^*}(w_M) - s_{Ww_M,f^*} + \sigma_{w_M,f^*} + s_{Ww_M,f^*} - t_{w_m \to m_E},$$

we obtain:

$$0 \geq \sigma_{w_M,f^*} + s_{Ww_M,f^*} - t_{w_m \to m_E},$$

and therefore,

$$t_{w_m \to m_E} \geq \sigma_{w_M,f^*} + s_{Ww_M,f^*} = u_{w_M}(\mu_W, \mathbf{s}_W).$$

■

The underlying intuition of Claim 6 is that the competition resulting from w_M seeking employment at an alternative firm cannot diminish the utility of existing workers by more than $t_{w_m \to m_E}$, given that the maximum utility that w_M can achieve under any stable allocation is lover than $t_{w_m \to m_E}$. To ensure that workers remain employed by a firm, their utility must remain positive after the introduction of the transfer. We decompose the transfer $\mathbf{T}$ using Lemma 1:

- $\mathbf{T}_1$ is defined such that for each $w \in m_E$ and $f \in F$, $t_{1_{w,f}} = t_{w,f}$, and 0 for other elements.
- $\mathbf{T}_2$ is defined such that $t_{2_{w_m,f^*}} = t_{w_m \to m_E}$, and 0 for other elements.
- $\mathbf{T}_3$ is defined such that $t_{3_{w_m,f^*}} = t_{w_m,f^*} - t_{w_m \to m_E}$, and 0 for other elements.

First, consider $(\mathbf{T}_0) \overset{\mathbf{T}_1}{\to} (\mathbf{T}_1)$. In this proof, we assume a strict inequality for the transfer with respect to employed minority workers. This assumption does not affect the argument and simplifies the proof's construction. Since for each $w \in m_E$, $t_{1_{w,\mu_W(w)}} = \max_{f \in F}(t_{1_{w,f}}) > \max(0, t_{w_m \to m_E} - (s_{Ww,\mu_W(w)} + \sigma_{w,\mu_W(w)}))$, it follows from Theorem 2 that

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) > t_{w_m \to m_E}.$$

Next, consider $(\mathbf{T}_1) \overset{\mathbf{T}_2}{\to} (\mathbf{T}_1 + \mathbf{T}_2)$. From Lemma 2 (ii), for each $w \in m_E$,

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) - u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) \leq t_{w_m \to m_E}.$$

[38] This decomposition reflects the redistribution of the surplus generated by w_M at f^*.

Hence,

$$u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) > 0.$$

In addition, Claim 5 implies that either $\mu_F^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f^*$, and by Lemma 3, it follows that $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f^*$.

Finally, consider $(\mathbf{T}_1 + \mathbf{T}_2) \xrightarrow{\mathbf{T}_3} (\mathbf{T})$, since $\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3$. We know that $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f^*$, and in $\mathbf{T}_3$, for each $w \in m_E^{\mathbf{T}_1+\mathbf{T}_2}$, $t_{3_{w,\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w)}} = \max_{f\in F}(t_{3_{w,f}})$, by Theorem 2, it follows that

$$u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \geq u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) + t_{3_{w,\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w)}} > 0.$$

Since for each $w \in m_E$, $u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) > 0$, we know that $\mu_W^{\mathbf{T}}(w) \neq \varnothing$. Furthermore, $\mu_W^{\mathbf{T}_1+\mathbf{T}_2} = \mu_W^{\mathbf{T}}$, and thus $\mu_W^{\mathbf{T}}(w_m) = f^*$. From Lemma 3, since $\mathbf{T}$ promotes the employment of minority workers under φ_F, it also promotes the employment of minority workers under any stable mechanism, which completes the proof. ■

D.9 Proof of Theorem 4

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, and $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$. Furthermore:

- For each $w \in m_E$, $t_{w,\mu_W(w)} = \max_{f\in F}(t_{w,f}) \geq t_{w_m \to m_E}$,
- For each $w' \in W \setminus (m_E \cup \{w_m\})$ and each $f \in F$, $t_{w',f} = 0$, and
- For each $f \in F \setminus \{f^*\}$, $t_{w_m,f} = 0$, and $t_{w_m,f^*} \geq t_{w_m \to m_E}$.

From Lemma 1, we decompose $\mathbf{T}$ into three transfers:

- $\mathbf{T}_1$ is defined such that for each $w \in m_E$ and each $f \in F$, $t_{1_{w,f}} = t_{w,f}$, and 0 for other elements.
- $\mathbf{T}_2$ is defined such that $t_{2_{w_m,f^*}} = t_{w_m \to m_E}$, and 0 for other elements.
- $\mathbf{T}_3$ is defined such that $t_{3_{w_m,f^*}} = t_{w_m,f^*} - t_{w_m \to m_E}$, and 0 for other elements.

First, consider $(\mathbf{T}_0) \xrightarrow{\mathbf{T}_1} (\mathbf{T}_1)$. Since for each $w \in m_E$, $t_{1_{w,\mu_W(w)}} = \max_{f\in F}(t_{1_{w,f}}) \geq t_{w_m \to m_E}$, it follows from Theorem 2 that

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) \geq u_w(\mu_W, \mathbf{s}_W) + t_{1_{w,\mu_W(w)}} \quad \text{for each } w \in m_E.$$

Next, consider $(\mathbf{T}_1) \xrightarrow{\mathbf{T}_2} (\mathbf{T}_1 + \mathbf{T}_2)$. From Lemma 2 (ii), for each $w \in m_E$,

$$u_w(\mu_W^{\mathbf{T}_1}, \mathbf{s}_W^{\mathbf{T}_1}) - u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) \leq t_{w_m \to m_E}.$$

Hence,

$$u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) \geq u_w(\mu_W, \mathbf{s}_W).$$

In addition, Claim 5 implies that $\mu_F^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f^*$, and by Lemma 3, it follows that $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f^*$.

Finally, consider $(\mathbf{T}_1 + \mathbf{T}_2) \xrightarrow{\mathbf{T}_3} (\mathbf{T})$, since $\mathbf{T} = \mathbf{T}_1 + \mathbf{T}_2 + \mathbf{T}_3$. We know that $\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w_m) = f^*$, and

in $\mathbf{T}_3$, for each $w \in m_E^{\mathbf{T}_1+\mathbf{T}_2}$, $t3_{w,\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w)} = \max_{f\in F}(t3_{w,f})$, by Theorem 2, it follows that

$$u_w(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) \geq u_w(\mu_W^{\mathbf{T}_1+\mathbf{T}_2}, \mathbf{s}_W^{\mathbf{T}_1+\mathbf{T}_2}) + t3_{w,\mu_W^{\mathbf{T}_1+\mathbf{T}_2}(w)} \geq u_w(\mu_W, \mathbf{s}_W).$$

Furthermore, $\mu_W^{\mathbf{T}_1+\mathbf{T}_2} = \mu_W^{\mathbf{T}}$, and thus $\mu_W^{\mathbf{T}}(w_m) = f^*$. From Lemma 3, since $\mathbf{T}$ promotes the employment of minority workers under φ_F, it also promotes the employment of minority workers under any stable mechanism, which completes the proof. ■

D.10 Proof of Proposition 6

Proof. Consider a market with capacities $\overline{G} \in \overline{\mathbb{G}}$, and let $\mathbf{T} \in \mathbb{T}$ be a transfer such that $t_{w,f} > 0$ only if $w \in m$, and $t_{w,f} \geq 0$ for all $w \in W$ and $f \in F$. Suppose there exists a firm $f \in F$ such that $|\mu_W^{\mathbf{T}}(f) \cap m| > |\mu_W(f) \cap m|$. Let $w^* \in m$ be a worker such that $\mu_W(w^*) \neq f$ and $\mu_W^{\mathbf{T}}(w^*) = f$ under any worker-optimal stable allocation. Let $f^* \equiv \mu_W(w^*)$.
By Lemma 4, we know that for each $f' \in F$ and for any pair of workers $w, w' \in \mu_W(f')$, the following equality holds:

$$R_{f'}(w) - s_{Ww,f'} = R_{f'}(w') - s_{Ww',f'}.$$

Since $(\mu_W, \mathbf{s}_W)$ is a worker-optimal stable allocation, it follows that:

$$u_{w^*}(\mu_W, \mathbf{s}_W) = R_{f^*}(w^*) + \sigma_{w^*,f^*} - (R_{f^*}(w) - s_{Ww,f^*}),$$

for any $w \in \mu_W(f^*)$.
Because $f \neq f^*$, it must be that

$$R_f(w') - s_{Ww',f} > R_f(w^*) + \sigma_{w^*,f} - u_{w^*}(\mu_W, \mathbf{s}_W). \tag{12}$$

for any $w' \in \mu_W(f)$. That is, at fixed utility, the profit generated by hiring w^* would be insufficient for firm f.
Now consider a worker-optimal stable allocation with $\mathbf{T}$, $(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$. By the same logic, we know that:

$$u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) = R_f(w^*) + t_{w^*,f} + \sigma_{w^*,f} - (R_f(w'') + t_{w'',f} - s_{Ww'',f}^{\mathbf{T}}), \tag{13}$$

for any $w'' \in \mu_W^{\mathbf{T}}(f)$.
Rewriting (13),

$$R_f(w'') + t_{w'',f} - s_{Ww'',f}^{\mathbf{T}} = R_f(w^*) + t_{w^*,f} + \sigma_{w^*,f} - u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}). \tag{14}$$

Since $(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$ is stable, it follows that the pair $\{f^*\} \cup \{w^*\}$ does not block the allocation at salary $s_{W,w^*,f^*}^{\mathbf{T}}$. We distinguish three cases.

- **Case 1:** $V_{f^*}(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_{f^*}, \mathbf{T}) > V_{f^*}(\mu_W; \mathbf{s}_W, R_{f^*}, \mathbf{T}_0)$, that is, firm f^*'s profit is strictly

higher than before the introduction of $\mathbf{T}$. In this case, it remains to show that

$$t_{w^*,f} > u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) - u_{w^*}(\mu_W, \mathbf{s}_W).$$

If

$$u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) < u_{w^*}(\mu_W, \mathbf{s}_W),$$

the result follows immediately since $t_{w^*,f} \geq 0$. Otherwise, note that the firm-worker set $\{f\} \cup \mu_W(f)$ does not block the allocation $(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$ at the salaries given by $\mathbf{s}_W$. Equation (12) therefore implies that firm f^*'s profit has (weakly) increased. Combining (12) and (14) yields

$$R_f(w^*) + t_{w^*,f} + \sigma_{w^*,f} - u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) > R_f(w^*) + \sigma_{w^*,f} - u_{w^*}(\mu_W, \mathbf{s}_W). \tag{15}$$

Simplifying (15),

$$t_{w^*,f} > u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) - u_{w^*}(\mu_W, \mathbf{s}_W). \tag{16}$$

- **Case 2:** $u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) > u_{w^*}(\mu_W, \mathbf{s}_W)$, that is, worker w^*'s utility has strictly increased. Since $(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$ is stable, the firm-worker set $\{f\} \cup \mu_W(f)$ does not block the allocation at the salaries $\mathbf{s}_W$. Therefore, firm f's profit has weakly increased under $\mathbf{T}$. Combining (12) and (14) as in Case 1 yields (15) and (16). Suppose, by contradiction, that

  $$V_f(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_f, \mathbf{T}) = V_f(\mu_W; \mathbf{s}_W, R_f, \mathbf{T}_0).$$

  Then the pair $\{f^*, w^*\}$ does not block $(\mu_W, \mathbf{s}_W)$, and yields the same joint surplus under both allocations. This implies the existence of two distinct stable allocations with different assignments.
  This contradicts Lemma 3, which guarantees the uniqueness of the assignment under genericity. Therefore,

  $$V_f(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_f, \mathbf{T}) > V_f(\mu_W; \mathbf{s}_W, R_f, \mathbf{T}_0).$$

- **Case 3:** $V_{f^*}(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_{f^*}, \mathbf{T}) = V_{f^*}(\mu_W; \mathbf{s}_W, R_{f^*}, \mathbf{T}_0)$ and $u_{w^*}(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}}) = u_{w^*}(\mu_W, \mathbf{s}_W)$, that is, both firm f^*'s profit and worker w^*'s utility are unchanged. In this case, it remains to show that $t_{w^*,f} > 0$ and that

  $$V_f(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_f, \mathbf{T}) > V_f(\mu_W; \mathbf{s}_W, R_f, \mathbf{T}_0).$$

  Since $\mu_W^{\mathbf{T}}(w^*) = f$, stability of $(\mu_W^{\mathbf{T}}, \mathbf{s}_W^{\mathbf{T}})$ implies that the firm-worker set $\{f\} \cup \mu_W(f)$ does not block the allocation at the salaries $\mathbf{s}_W$. Equation (12) therefore implies that firm f^*'s profit has (weakly) increased under $\mathbf{T}$. Combining (12) and (14) as in Case 1 yields (15) and (16). Since utilities are unchanged in this case, it follows that $t_{w^*,f} > 0$.
  Suppose, by contradiction, that

  $$V_f(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_f, \mathbf{T}) = V_f(\mu_W; \mathbf{s}_W, R_f, \mathbf{T}_0).$$

Then the pair $\{f^*, w^*\}$ does not block $(\mu_W, \mathbf{s}_W)$, and yields the same joint surplus under both allocations. This implies the existence of two distinct stable allocations with different assignments.

This contradicts Lemma 3, which guarantees the uniqueness of the assignment under genericity. Therefore,

$$V_f(\mu_W^{\mathbf{T}}; \mathbf{s}_W^{\mathbf{T}}, R_f, \mathbf{T}) > V_f(\mu_W; \mathbf{s}_W, R_f, \mathbf{T}_0),$$

which completes the proof.

■

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